HYDRAULIC MODELS AND DIMENSIONAL ANALYSIS

WATER RESOURCES DEVELOPMENT TRAINING CENTRE

Nayan Sarma, B.E.
1977

HYDRAULIC MODELS AND DIMENSIONAL ANALYSIS

BHARAT SINGH,
C.E. (Hons.) Roorkee, Ph.D. (London), D.I.C.
Professor Designs (Civil)

WATER RESOURCES DEVELOPMENT TRAINING CENTRE
AT UNIVERSITY OF ROORKEE
INDIA

Price: Rs.6.00

1973

Printed and Published by
Directorate Technical Publications
At University of Roorkee
ROORKEE

C O N T E N T S

...o....

CHAPTER-1

PRINCIPLES OF HYDRAULIC SIMILITUDE

1.1. INTRODUCTION:

Knowledge of principles of similitude is essential for experimental work in Hydraulics, whether for fundamental studies in fluid flow or for testing the designs of Hydraulic Structures.

The laws of similitude are used by the engineer not only in connection with the design of ships, aircraft, and various hydraulic structures, but in the analysis, as well, of various motion occurrences of both fluid and solid media by comparison with other fluid or solid media- e.g. by a knowledge of principles of similitude, it is possible to analyse flow conditions of gas or oil in a closed conduit from a knowledge of flow of water in another conduit or vice-versa. Though perfect similarity is seldom (Rarely) obtainable, yet within the usual requirements of accuracy, similarity is obtainable in most cases.

1.2. PHYSICAL SIGNIFICANCE OF THE MECHANICS OF SIMILITUDE:

There can be different aspects of similarity. In particular, one may refer to (i) Geometrical similarity (ii) Kinematic similarity (iii) Dynamic or Mechanical similarity.

Geometrical Similarity: Two objects are said to be geometrically similar if the ratios of all homologous dimensions are equal. Thus geometric similarity involves only similarity in form.

Kinematic Similarity: This is similarity of motion. Two motion occurrences are kinematically similar if the patterns of paths of motion are geometrically similar, and if the ratios of velocities of the various homologous particles involved in the motion occurrences are equal.

Dynamic Similarity: This is similarity of masses and forces. Two motion occurrences are dynamically similar if they are-

(a) kinematically similar,
(b) the ratios of the masses of the various homologous objects involved in the motion occurrences are equal, and if
(c) the ratios of the homologous forces which in anyway affect the motion of the homologous objects are equal.

According to Newton's second law, force equals mass times acceleration. Thus for dynamic similarity to obtain between the two similar systems, the ratio of the forces acting on any two homologous mass particles must, be proportional to the product of the respective homologous masses and their corresponding accelerations.

Ordinarily, the resultant accelerating force is composed of a system of various forces, although one or more of these frequently predominates. In fluid motion phenomena, forces and physical properties influencing motion occurrences include primarily the following:

(i) earth's gravitational forces
(ii) friction between fluid particles
(iii) capillarity
(iv) elastic forces, or compressive forces

The analytical treatment of mechanics of similitude is primarily based on Newton's second law of motion, $F = ma$.

Consider two geometrically similar systems, one large and one small, which will be referred to respectively as the 'prototype' and the 'model'. The systems may be made up of fluid or solid media, each of definite physical characteristics - e.g. the flow past an obstruction in a river such as a bridge pier. The media in the prototype and model may or may not be the same, concievably water may be the fluid in the prototype and kerosene that in the model.

Movement of the respectively homologous particles is considered in an arbitrarily chosen direction x, so that the resultant forces producing motion are as follows:-

$$F_p = m_p \frac{d^2 x_p}{d\,(T_p)^2} \qquad \text{(for the prototype)} \quad (1)$$

and $$F_m = m_m \frac{d^2 x_m}{d\,(T_m)^2} \qquad \text{(for the model)} \quad (2)$$

Where,

x_p, x_m are homologous distances,

T_p, T_m are homologous time periods,

F_p, F_m are homologous forces, and

m_p, m_m are homologous masses.

For geometrical similarity, the ratio of all homologous linear dimensions must be the same, thus

$$\boxed{\frac{x_m}{x_p} = L_r} \qquad (3)$$

For kinematic similarity the ratio of the time periods required for any two homologous particles to travel homologous paths must be constant throughout the system, so that

$$\boxed{\frac{T_m}{T_p} = T_r} \qquad (4)$$

Lastly, for dynamic similarity the ratio of the masses of homologous particles, when in corresponding positions, must be constant for all mass particles throughout the system, and homologous mass accelerating forces must be constant, or,

$$\frac{m_m}{m_p} = m_r \tag{5}$$

and

$$\frac{F_m}{F_p} = F_r \tag{6}$$

1.3. DERIVED RATIOS:

Area ratio - $\dfrac{A_m}{A_p} = A_r = (L_r)^2$ (7)

Volume ratio- $\dfrac{V_m}{V_p} = V_r = (L_r)^3$ (8)

Since mass is density times volume, and density $\rho = \frac{\gamma}{g}$ it follows that

$$m_r = \frac{m_m}{m_p} = \frac{\rho_m . \ V_m}{\rho_p \ \ V_p} = \frac{[\gamma/g \ \ V]_m}{[\gamma/g \ \ V]_p} =$$

$$m_r = \left[\frac{\gamma}{g}\right]_r \left[L^3\right]_r \tag{9}$$

The velocity of a particle may be represented by

$$V = \frac{dL}{dT}$$

Hence $V_r = \dfrac{V_m}{V_p} = \dfrac{d \ x_m}{d \ T_m} \Big/ \dfrac{d \ x_p}{d \ T_p} = \dfrac{L_r}{T_r}$ (10)

Similarly acceleration $a = \dfrac{dV}{dT} = \dfrac{d^2x}{d \ T^2}$

Hence $a_r = \dfrac{L_r}{(T_r)^2}$ (11)

Taking advantage of these derived ratios, the ratio of the accelerating forces given by equations (1) and (11) can be written as -

$$F_r = m_r \ \frac{L_r}{(T_r)^2} \tag{12}$$

These two interdependent equations express Newton's general law of similitude. They are a direct consequence of inertia, since they are derived directly from the dynamic equation F = ma, with the qualification that the prototype and model occurrences shall be dynamically similar.

In the application of equations (15) to practical problems, K may be constant over wide range of conditions. E.g. the drag coefficient at high Reynolds nos. is independent of the Reynolds no. On the other hand at low Reynolds numbers it is a function of the Reynolds number. If model testing is done in the range of Reynolds numbers at which the drag coefficient is a function of the Reynolds number, the results will not be applicable directly to the prototype.

1.5. SPECIAL MODEL LAWS:

Equation (14) is a general condition which has to be fulfilled in all cases. It gives the force ratio in terms of length and time ratios. A second equation connecting the same variables, arises from the special nature of the occurrence. These two equations give us a relation between time and length ratios applicable to that occurrence.

If two forces are equally important in an occurrence we shall have to write a third equation (the first relating inertial forces, the second the primary force are always necessary) for this second force. Thus there will be three equations for three variables F_r, L_r and T_r. In such a case there would be only one solution. None of the ratios could be arbitrarily fixed. If the two media in the model and the prototype are the same, similarity would be obtained only if the $L_r = 1$, i.e. model is of the same size as the prototype. In the case of different media, the scale ratios will be fixed by the properties of the media.

Though more than one force are involved in many flow problems, usually the effect of one of them becomes significant only beyond certain critical conditions. Thus if the model is operated outside those critical conditions, the motion occurrences are sufficiently similar for practical utilisation of model results.

1.6. THE FROUDE LAW:

Consider two occurrences in which gravity is the only force producing motion. Other forces, such as fluid friction, surface tension etc. are negligible. With certain limitations, this would cover the case of flow over weirs and spillways or through sluices and orifices, propagation of gravity waves etc.

Now, to satisfy the general inertial equation (13)

$$F_r = \rho_r \; (L_r)^3 \; \frac{L_r}{(T_r)^2}$$

Equation (12) reqpresents the force ratio in terms of length and time ratios. It is a necessary condition for dynamic similarity between two motion occurrences and is sometimes referred to as Bertrand qualifying equation. Several forces may be involved in a given phenomenon in making up the resultant force though one of them is usually dominant. If the occurrence be such that several equally important forces produce the motion, the problem of attaining similitude becomes more involved. Similitude can be secured, when two forces exist, by the use of different fluids in model and prototype. Once the two fluids are chosen, the scale ratio becomes fixed by the properties of the chosen fluids.

In general practice, however, the concern of the investigator is with the effect of only one force which is dominant. The neglect of the other forces is responsible for inaccuracies only in the final result. The endeavour therefore, should be to chose scales and to build and operate models in such a manner that the effect of non-dominant forces is compensating or negligible - e.g. in a phenomenon in which gravity is predominant but fluid friction and surface tension may also be involved, their effect can be minimised by avoiding small size of the model.

1.4. NEWTON'S GENERAL LAW OF SIMILITUDE:

If in two dynamically similar occurrences, such as a prototype and model, homologous mass particles, m_m and m_p experience homologous accelerations a_m and a_p respectively, the two masses require accelerating forces $F_m = m_m . a_m$ and $F_p = m_p . a_p$ to over-come inertia. Equation(12) can be written in the form:

$$F_r = \rho_r (L_r)^3 \frac{L_r}{(T_r)^2} \qquad (13)$$

$$\text{or} \quad F_r = \rho_r (L_r)^2 \frac{(L_r)^2}{(T_2)^2}$$

$$= \rho_r A_r (V_r)^2 \qquad (14)$$

If the constant of proportionality in the force equations is taken as k, the inertial forces of homologous particles in the model and the prototype, respectively, may be written as :-

$$F_m = K \rho_m A_m (V_m)^2 \qquad (15a)$$

$$\text{and} \quad F_p = K \rho_p A_p (V_p)^2 \qquad (15b)$$

The gravitation force ratio will be equal to the ratio of the weights of homologous particles, so that for the second equation -

$$F_r = \frac{W_m}{W_p} = \frac{\gamma_m}{\gamma_p} \left(\frac{L_m}{L_p}\right)^3 = \gamma_r (L_r)^3 \qquad (16)$$

equating the two values of F_r from equation (13) and (16)

$$\frac{\rho_r (L_r)^4}{T_r^{\,2}} = \gamma_r \; L_r^{\,3}$$

and since $\rho = \frac{\gamma}{g}$

$$T_r = \sqrt{\frac{L_r}{g_r}} \qquad (17)$$

Also $V_r = \frac{L_r}{T_r} = \sqrt{g_r L_r}$ (18)

Since g_r is always unity in practice,

$$T_r = \sqrt{L_r} \qquad (17a)$$

and $V_r = \sqrt{L_r}$ (18a)

A large proportion of the phenomenon encountered in hydraulic laboratory practice are controlled by gravity and for all of these, relations 17(a) and 18(a) are important.

In dimension less form, Newton's general law of similitude can be written as -

$$\frac{F_M}{\rho_m (L_m)^2 (V_m)^2} = \frac{F_p}{\rho_p (L_p)^2 (V_p)^2}$$

This relation is true, regardless of the system of units used so long as they are consistent, among themselves.

Consider Froude's model law,

$$V_r = \sqrt{g_r L_r} \qquad \text{eqn. (18)}$$

or

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

which means that the dimensionless parameter $\frac{V}{\sqrt{gL}}$, called the Froude's no., should have the same value in the model and the prototype for dynamic similarity in gravitational phenomenon.

1.7. THE REYNOLD'S LAW:

Consider the motion occurrences in two incompressible liquids taking place in a similar fashion under the general influence of internal friction forces; disturbing secondary influences are assumed to be unimportant. Such a state of motion would obtain, for example, in flow through small pipes; flow around immersed bodies at subsonic velocities, etc.

The resistance to motion is dependent upon the viscosity of the fluid. Thus if U indicates the velocity of the fluid at any point in the direction of motion and A, the surface area between adjoining lamina of fluid having uniform relative velocities, then,

$$F = \mu \frac{du}{dy} A \tag{19}$$

in which F is the frictional resistance and μ is the dynamic viscosity. The expression $\frac{du}{dy}$ indicates rate of change of velocity in the direction normal to u.

Again equating the inertial force ratio given by equation (13), to the force ratio given by equation(19).

$$F_r = \mu_m \frac{du_m}{dy_m} A_m \div \mu_p \frac{du_p}{dy_p} A_p = \frac{\mu_m}{\mu_p} \frac{L_r}{T_r} \cdot \frac{1}{L_r} L_r^2$$

$$= \mu_r \frac{L_r^2}{T_r}$$

Hence $\rho_r \frac{L_r^4}{T_r^2} = \mu_r \frac{L_r^2}{T_r}$

$$T_r = L_r^2 \cdot \frac{\rho_r}{\mu_r}$$

or $T_r = \frac{(L_r)^2}{\nu_r}$ ν_r = Kinematic viscosity (20)

which is the Reynold's model law for Time ratios. If the same fluid is used in model and prototype, ν_r is unity and $T_r = L_r^2$

Again from equation (20)

$$T_r = \frac{(L_r)^2}{\nu_r}$$

or $\frac{L_r}{T_r} \frac{L_r}{\nu_r} = 1$, or $\frac{V_r L_r}{\nu_r} = 1$ (21)

Since $\frac{VL}{\nu}$ denotes the Reynolds no., it follows that for dynamic similarity in phenomenon governed by fluid friction, the Reynold's no. in the model and the prototype should be equal.

1.8. THE WEBER LAW :

At the free surface of a liquid, and also at the surface of separation between two liquids, there can be observed what behaves as an elastic skin. This is called surface tension and is measured in force per unit length of periphery. It is usually denoted by σ

Surface tension phenomenon governs capillary travel of fluids and formation of bubbles and drops and capillary waves. Surface tension decreases with increa e in temperature. At the boundary between water and air, the force of surface tension is 75 dynes/cm. or 5.0×10^{-3} lbs/ft. at 70°F.

Surface tension force,

$$F = \sigma L \qquad (22)$$

and $$F_r = \sigma_r L_r \qquad (23)$$

Equating this to inertial force ratio,

$$\rho_r \frac{(L_r)^4}{(T_r)^2} = \sigma_r L_r$$

from which $$T_r = (L_r)^3 \rho_r/\sigma_r \qquad (24)$$

Or $$T_r = \frac{L_r^{3/2} \rho_r^{1/2}}{\sigma_r^{1/2}}$$

Or $$T_r^2 = \frac{L_r^3 \rho_r}{\sigma_r}$$

Or $$\left(\frac{L_r}{T_r}\right)^2 \cdot \frac{L_r}{\sigma_r/\rho_r} = 1$$

Or $$\frac{V_r^2 L_r}{\omega_r} = 1 \qquad (25)$$

where $\omega_r = \frac{\sigma_r}{\rho_r}$ and may be designated the kinematic surface tension. $\frac{V_r^2 L_r}{\omega_r}$ is called the 'weber number'

and for dynamic similarity in phenomenon involving surface tension as the predominant force, this number should be equal in the model and the prototype.

1.9. THE CAUCHY LAW:

The bulk modulus of elasticity of a liquid is similar to that of a solid, in that it denotes the ratio of an increment in stress to a decrement in volume caused thereby. A decrement in volume represents an increment in density. Hence the bulk modulus is given by -

$$K = \frac{\Delta p}{-\frac{\Delta V}{V}} = \frac{\Delta p}{\Delta \rho / \rho}$$

or $$K = \rho \frac{\Delta p}{\Delta \rho} \qquad (26)$$

in which K represents a pressure intensity. The force involved is expressed as the product of this pressure intensity, and the area over which it is applied.

Hence the ratio of forces between the model and the prototype, (for the same volumetric strain or percentage change in volume)

$$F_r = K_r \; (L_r)^2 \qquad (27)$$

Equating this force ratio to the inertia ratio,

$$K_r \, (L_r)^2 = \rho_r \, \frac{(L_r)^4}{(T_r)^2}$$

Hence, $$T_r = \frac{L_r}{\sqrt{K_r / \rho_r}}$$

or, $$T_r = \frac{L_r}{\sqrt{e_r}}$$

in which e is the kinematic elasticity $= \frac{K}{\rho}$

For dynamic similarity where elastic forces are dominant.

$$\frac{L_r}{T_r \sqrt{e_r}} = 1$$

or $$\frac{U_r}{\sqrt{e_r}} = \text{'C'} = 1$$

i.e. the Cauchy number $\frac{U}{\sqrt{e}}$ should have the same value in the model and the prototype.

EXAMPLES:

1. A Froude's law model is made at a length scale of 1/50. Find the time velocity, discharge and force scales.

As the same fluid is used, and gravity is not alterable, the scale ratios are determined as below:

$$T_r = L_r^{\frac{1}{2}} = \frac{1}{\sqrt{50}} = \frac{1}{7.1}$$

$$V_r = L_r^{\frac{1}{2}} = \frac{1}{\sqrt{50}} = \frac{1}{7.1}$$

$$Q_r = L_r^{5/2} = \left(\frac{1}{50}\right)^{5/2} = \frac{1}{17,750}$$

$$F_r = L_r^{3} = \left(\frac{1}{50}\right)^{3} = \frac{1}{125,000}$$

2. A model of a short length tunnel spillway should satisfy both Froude's and Reynold's law for true similarity. Find the required condition.

Equate the velocity scales for Froude and Reynolds law (See Table 1.1)

$$L_r^{\frac{1}{2}} \; g_r^{\frac{1}{2}} = \frac{\mu_r}{L_r \rho_r} = \frac{\nu_r}{L_r}$$

or $$L_r^{3/2} \; g_r^{\frac{1}{2}} = \nu_r$$

Since g_r has to remain unity, this requires use of a fluid such that $\nu_r = L_r^{3/2}$. For example in a 1/50 scale model the fluid used should be such that its kinematic viscosity is 1/355 that of prototype or water. In practice it is impossible to find and use such a fluid.

3. Priming time of a siphon spillway depends on gravity as well as surface tension. Find the condition for simultaneous satisfaction of both laws.

Again equating velocity scales for Froude and Weber laws, we get

$$L_r^{\frac{1}{2}} \; g_r^{\frac{1}{2}} = \left(\frac{\sigma_r}{L_r \rho_r}\right)^{\frac{1}{2}}$$

or $$L_r \; g_r = \frac{\sigma_r}{L_r \rho_r}$$

Since g_r remains unity,

$$\left\{\frac{\sigma}{\rho}\right\}_r = L_r^{2}$$

The kinematic surface tension should be reduced in proportion to L_r^2. It is possible to reduce surface tension with the help of detergents to a certain extent.

TABLE: 1.1

FLUID PROPERTY SCALES

Characteristic 1	Dimension 2	SCALE RATIOS FOR THE LAWS OF Froude 3	Reynolds 4	Weber 5	Cauchy 6
(a) Physical Properties					
Force characteristics		Gravity	Viscosity	Surface tension	Elasticity
Length	L	L_r	L_r	L_r	L_r
Area	L^2	$(L_r)^2$	$(L_r)^2$	$(L_r)^2$	$(L_r)^2$
Volume	L^3	$(L_r)^3$	$(L_r)^3$	$(L_r)^3$	$(L_r)^3$
(b) Kinematic Properties					
Time	T	$\sqrt{(L\rho/\gamma)_r}$	$(L^2\rho/\mu)_r$	$\sqrt{(L^2\rho/\sigma)_r}$	$(L\sqrt{\rho/K})_r$
Velocity	LT^{-1}	$\sqrt{(L\gamma/\rho)_r}$	$(\mu/L\rho)_r$	$(\sigma/L\rho)_r$	$(K/\rho)_r$
Acceleration	LT^{-2}	(γ/ρ)	$(\mu^2/\rho^2L^3)_r$	$(\sigma/L^2\rho)_r$	$(K/L\rho)_r$
Discharge	L^3T^{-1}	$(L^{5/2}\sqrt{\gamma/\rho})_r$	$(L\mu/\rho)_r$	$(L^{3/2}/\sqrt{\sigma/\rho})_r$	$(L\sqrt{K/\rho})_r$

(c) <u>Dynamic Properties</u>

1	2	3	4	5	6
Mass	M	$(L^3 \rho)_r$	$(L^3 \rho)_r$	$(L^3 \rho)_r$	$(L^3 \rho)_r$
Force	MLT^{-2}	$(L^3 \gamma)_r$	$(\mu^2/\rho)_r$	$(L \sigma)_r$	$(L^2 K)_r$
Density	ML^{-3}	ρ_r	ρ_r	ρ_r	ρ_r
Specific weight	$ML^{-2}T^{-2}$	γ_r	$(\mu^2/L^3 \rho)_r$	$(\sigma/L^2)_r$	$(K/L)_r$
Dynamic viscosity	$ML^{-1}T^{-1}$	$(L^{3/2}\sqrt{\rho\gamma})_r$	μ_r	$(L \rho \sigma)_r^{1/2}$	$(L\sqrt{K \rho})_r$
Surface tension	ML^{-1}	$(L^2 \gamma)_r$	$(\mu^2/L \rho)_r$	σ_r	$(L/K)_r$
Volume elasticity	$ML^{-1} T^{-2}$	$(L \gamma)_r$	$(\mu^2/L^2 \rho)_r$	$(\sigma/L)_r$	K_r
Pressure intensity	$ML^{-1}T^{-2}$	$(L \gamma)_r$	$(\mu^2/L^2 \rho)_r$	$(\sigma/L)_r$	K_r
Momentum impulse	MLT^{-1}	$(L^{7/2}\sqrt{\rho\gamma})_r$	$(L^2 \mu)_r$	$(L^{5/2}\sqrt{\rho\sigma})_r$	$(L^3\sqrt{K \rho})_r$
Energy and work	ML^2T^{-2}	$(L^4 \gamma)_r$	$(L \mu^2/\rho)_r$	$(L^2 \sigma)_r$	$(L^3 K)_r$
Power	ML^2T^{-3}	$(L^{7/2}\gamma^{3/2}/\rho^{1/2})_r$	$(\mu^3/L \rho^2)_r$	$(\sigma^{3/2}\sqrt{L/\rho})_r$	$(L^2 K^{3/2}/\rho^{1/2})_r$

Note: In gravity scales $\gamma = g \rho$ and $(\gamma/\rho)_r = g_r$ can not be altered.

CHAPTER-II

DIMENSIONAL ANALYSIS

2.1. Definitions(a) Physical Entities- There exist in the physical world, certain things which we call "Physical entities". Thus length, time, velocity, friction, electric current, heat, etc. are physical entities.

(b) Physical quantities - Physical entities are subject to measurement. The measured entities are called physical quantities.

(c) Physical Units- In order to measure a physical entity it is necessary to have a standard or measure. This standard is called a "Physical Unit". Thus cm, in., minute., miles/hr., dynes, amperes, calories are physical units.

2.2. Choice of Fundamental and Derived Entities:

As our knowledge of laws of the physical world expands we discover new entities, as well as relationships between already existing entities. For example Newton's laws relate mass and force. The phenomenon of atomic fission brought new physical entities into the science of Physics. Einstein correlated mass to energy or work.

The existence of these relationships has resulted in the division of physical entities into two classes called 'fundamental' and 'derived' entities.

Fundamental(Primary) entities are the minimum number of physical entities in terms of which all other entities may be expressed.

Derived (secondary) entities are all physical entities other than the Fundamental(Primary) ones.

In problems of mechanics and hydro-dynamics, three fundamental entities are required. These could be chosen in more than one way. The most commonly used system is Mass, Length and Time. Force, Length, and time are also occassionally used.

In terms of the first system, velocity is $\frac{\text{length}}{\text{Time}}$ acceleration, i.e. $\frac{\text{change in velocity}}{\text{interval of time}}$ is $\frac{\text{length}}{\text{Time}^2}$ and so on. These are derived entities.

The relationship between fundamental and derived entities is independent of the units used. It is an algebraic relationship and holds true irrespective of units.

2.3. Dimensional symbolism and dimensional formulae:

Dimensional relations are not quantitative, and only express relationships between entities. To distinguish them from quantitative relationships, dimensional symbols are written in letters enclosed within brackets. Thus time is represented dimensionally by the symbol (T), force by (F) and so on.

The dimensional formula of a physical entity is the formula showing the relationship of the physical entity to the fundamental entities. E.g. the following dimensional formulae may be noted:-

$$[V] = [LT^{-1}]$$

$$[F] = [Ma] = [MLT^{-2}]$$

The physical dimensions of a physical entity are the exponents of the fundamental entities in its dimensional formula. E.g. in terms of L, M, and T, the dimensions of acceleration are 1 for length, 0 for M and -2 for T, and those of force for L, 1 for M and -2 for T.

The dimensional formula as well as the dimensions of a physical entity depend upon the choice of the fundamental entities. The adoption of one system in preference to another depends on the convenience of the user.

2.4. Postulates in dimensions:

(a) In any dimensional system, the formula of a physical entity in terms of the fundamental entities consists of products of powers of these entities (Table 2.1). In a physical equation involving sums and products of various physical entities, each term of the equation must have the same dimension, otherwise the various terms could not be added or subtracted. If would be absurd to add three kilogrammes to four metres ! This leads us to the first important postulate on dimensions -

'All terms of a physical equation must have identical dimensions'.

This means that the exponents of all the fundamental entities must be identical in all the terms of a physical equation. If this criterion is not satisfied, there is some thing wrong with the derivation of the equation, E.g. the critical tractive stress equation,

$$\tau_{cr} = .047 \ (\gamma - \gamma_w) \ D$$

satisfies this criterion. So does Stoke's law for terminal fall velocity 'w' of a sphere,

$$w = \frac{\gamma_s - \gamma_w}{18 \mu} \ D^2$$

If some one were to state, or to derive, these equations such that the dimensions of the terms on both sides, or of different terms on the same side, were not equal, the equations would be definitely wrong.

It may be noted that certain emperical physical relationships (not derived), are some times observed and stated in a particular system of units, such that the dimensions of the entities on the two sides are not the same e.g. consider Lacey's perimeter equation in f.p.s. units -

$$P = 8/3\ Q^{1/2}$$

On the left hand side, the perimeter has dimension 1 for length, 0 for mass and 0 for time. On the right hand side, the dimensions of $Q^{1/2}$ are $L^{3/2}\ T^{-1/2}$. Thus the dimensions on the two sides are not the same. In such equations, the numerical constant is not a pure number, but involves unknown physical entities. Its value will change if the equation were to be expressed in a different set of units, which would not have happened if the equation were dimensionally homogeneous.

It is a much better practice to express even emperical relationships in a dimensionally homogeneous form. In this way they are more likely to be complete relationships, and will also be equally applicable in all sets of units.

It must be pointed out that the satisfaction of postulate 'A' does not guarantee the accuracy of the derivation - it is a necessary but not a sufficient condition for accuracy of derivation of an ecuation between physical entities.

Postulate -b- A glance at Table 2.1 shows that all powers to which physical entities are raised are pure numbers and not other physical entities. A number is not a physical entity and, therefore, has no dimensions. Thus the second postulate follows as below :-

'All exponents of physical entities are dimensionless numbers and all exponents in a physical equation are dimensionless numbers,

E.g. the exponent in the suspended load equation

$$\frac{C}{C_a} = \left[\frac{d-y}{y} \quad \frac{a}{d-a}\right]^{w/kU_*}$$

$\frac{w}{kU_*}$ is a dimensionless number.

Where:

C = concentration at any height, if

Ca = given concentration at height a

d = total depth of the channel

w = fall velocity of particle

k = Von Karman's universal coefficient

U = shear velocity

A corollary which follows directly from this is —

All arguments of functions appearing in a physical equation are dimensionless.

E.g. consider the displacement in a simple harmonic motion,

$x = a \sin \omega t$

ω is the angular velocity equal to $\frac{2\pi}{T}$ where T is the time period of one cycle. Thus $\omega t = \frac{2\pi t}{T}$, and is dimensionless.

Again consider Dupuit's equation for discharge from a confined aquifer,

$$Q = \frac{2 K \Delta h y}{\log_e \frac{r_2}{r_1}}$$ (Δh loss of head between r_2 and r_1, y thickness of aquifer)

$\frac{r_2}{r_1}$ is a dimensionless ratio.

In all correctly derived physical equations, these postulates will hold true.

2.5. Derivation of simple physical laws:

It has been pointed out earlier that the dimensions of derived physical quantities are deduced either from definitions or from the applications of known physical laws. Conversely, unknown physical laws may be derived by the use of postulates of physical dimensions given above.

The first step in the dimensional analysis of a problem is to decide what variables enter the problem. If variables are introduced that really do not affect the phenomenon, too many terms may appear in the final equation. If variables are omitted that logically may influence the phenomenon, the calculations may reach an impasse or lead to an incomplete of erroneous result. Even though some variables are practically constants (e.g. the acceleration due to gravity) they may be essential because they combine with other active variables to form dimensionless products. It thus follows that before one undertakes the dimensional analysis of a problem, he should try to form a theory of the mechanism of the phenomenon - he must understand enough about the problem to explain why and how the variables influence the phenomenon.

The direct method of dimensional analysis, sometimes called Reyleigh's method is illustrated below with the help of a few examples.

Example (1) - Experience shows that the period of oscillation, T, of a simple pendulum depends on its length. It is also inferred that it depends on gravitational acceleration, g.

Write the relation as follows :

$$[T] = [L^a]\,[g^b]$$

substituting the dimensional formulae,

$$(T) = (L^a)\;(LT^{-2})^b$$

Equating the exponents of L and T on both sides,

$$a + b = 0$$
$$-2b = 1$$

Whence $a = 1/2$ and $b = -1/2$

Thus $T \propto \sqrt{\frac{l}{g}}$

Suppose we had assumed T to be a function of the mass of the pendulum bob. We would then get the dimensions of M equal to zero, and in this case, the error would be corrected by dimensional analysis itself. But it would not be so in all cases.

Example (2) Discharge over a rectangular weir with suppressed end contractions.

$$Q = \text{Constt}\;\; L^a \;\; g^b \;\; H^c$$

Where L is the length of crest, g the accn. due to gravity, and H the head over crest.

Writing the dimensional equation

$$[L^3\,T^{-1}] = [L^a]\,[LT^{-2}]^b\,[L]^c$$

Equating the exponents,

$$3 = a + b + c$$
$$-1 = -2b,\quad b = 1/2$$

Hence $a + c = 5/2$

Here, the dimensions of mass are zero on both sides, and we have been able to get only two equations for three unknown. Thus algebraically, complete solution is not possible. However, physically one can see that discharge must be directly proportional to length of crest, so that $a = 1$. So that we get $C = 3/2$

and $Q \propto \sqrt{g}\;\; LH^{3/2}$

Example-3: Since there are three fundamental entities in mechanical problems, three equations can be written for dimensional homogeniety. When more than three variable, are involved, a complete solution is not possible but often useful results can be obtained by a little ingenuity. Consider the problem of drag force exercised by a fluid on a sphere of diameter, D. We assume that the functional relationship is of the form,

$$F = kV^a D^b \rho^c \mu^d$$

V = velocity
D = diameter
ρ = density of fluid
and μ = viscosity of fluid

The dimensional equation is,

$$[MLT^{-2}] = [LT^{-1}]^a [L]^b [ML^{-3}]^c [ML^{-1}T^{-1}]^d$$

Equating the exponents on the two sides,

for mass $1 = c + d$

for length, $1 = a + b - 3c - d$

for time $-2 = -a - d$

Solving in terms of a,

$d = (2 - a)$

$c = 1 - d = (a - 1)$

$b = 1 - a + 3c + d$
$= a$

Substituting these values,

$$F = kV^a D^a \rho^{a-1} \mu^{2-a}$$

$$= k \left(\frac{VD}{\mu}\right)^{a-2} \rho V^2 D^2$$

$$= k (R_n)^n \rho V^2 D^2$$

It is, however, unlikely that the form could have been obtained by one unfamiliar with the dimensionless Reynolds no. For a large number of variables, the use of Buchhkingham's π theorem is more systematic and convenient.

TABLE 2.1

DIMENSIONS OF SOME IMPORTANT ENTITIES

Entity	Symbol	Dimensional Formulae L.M.T. System	L.F.T. System
Length	L	L	L
Area	A	L^2	L^2
Volume	V	L^3	L^3
Angle	α	L^o	L^o
Curvature	c	L^{-1}	L^{-1}
Time	T	T	T
Frequency	f	T^{-1}	T^{-1}
Velocity	v	LT^{-1}	LT^{-1}
Acceleration	a	LT^{-2}	LT^{-2}
Angular velocity	v_o	T^{-1}	T^{-1}
Angular acceleration	a_o	T^{-2}	T^{-2}
Mass	M	M	$FL^{-1}T^2$
Density	ρ	ML^{-3}	$FL^{-4}T^2$
Force	F	MLT^{-2}	F
Pressure (Stress)	p,s	$ML^{-1}\ T^{-2}$	FL^{-2}
Momentum	m_o	MLT^{-1}	FT
Moment of momentum	m_m	ML^2T^{-1}	FTL
Energy(Work)	W	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Torque	τ	ML^2T^{-2}	FL
Efficiency	η	1	1
Strain	s_o	1	1

Contd.

Entity	Symbol	Dimensional Formulae	
		L.M.T. system	L.F.T. system
Modulus of elasticity	E_s	$ML^{-1}\ T^{-2}$	FL^{-2}
Dynamic Viscosity	μ	$ML^{-1}T^{-1}$	$FL^{-2}T$
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Compressibility	β	$M^{-1}LT^2$	$F^{-1}L^2$
Capillary constant (Surface tension)	σ	MT^{-2}	FL^{-1}
Gravitation constant	G	$M^{-1}L^3T^{-2}$	$F^{-1}L^4T^{-4}$

CHAPTER-III

DIMENSIONAL ANALYSIS-APPLICATIONS OF BUCKINGHAM'S THEOREM

3.1. Buckingham's Theorem:

The principal tool of dimensional analysis by means of which one accomplishes the organisation of the variables, is known as the π -theorem, first brought emphatically before the engineering profession by Buckingham in 1915 (TASME Vol.37, 1915). The essence of this theorem is as follows:-

If any variable A_1 depends upon the independent variables $A_2, A_3 \ldots\ldots\ldots A_n$, and upon no others, the general functional relationship may be written in the form,

$$A_1 = f\,(A_2, A_3 \ldots\ldots\ldots\ldots A_n)$$

These may also be grouped in another functional relationship equal to zero.

$$f'\,(A_1, A_2, A_3 \ldots\ldots\ldots\ldots A_n) = 0$$

The theorem then states that if all of these n variables may be described with m fundamental dimensional units, they may then be grouped into (n-m) dimensionless π terms:

$$\phi\,(\pi_1, \pi_2, \quad \pi_{n-m}) = 0$$

In each term there will be (m+ 1) variables, only one of which need be changed from term to term.

The variables of which each π terms is composed must evidently appear in such exponential form that every term will be truely dimensionless.

Though the theorem as stated by Buckingham holds in most cases it is not rigerously correct. The statement which can be mathematically proved is as follows:-

'The number of independent dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix'.

In order to understand the meaning of this statement, it would be necessary to briefly review the properties of determinants and matrices.

3.2. Determinants:

An n'th order determinant is a square array of n^2 numbers, to which a value is attached in a definite manner. A second order determinant is evaluated as follows:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Let a_{rc} be the number in the r^{th} row and the c^{th} column of an n^{th} orders determinant. An (n-1) order deteri-meant may be formed by crossing out the r^{th} row and the c^{th} column of the given determinant The product of this (n-1) order determinant with $(-1)^{r+c}$ is called the 'co-factor' of the element a_{rc}. With this definition, the following important theorem may be expressed -

The sum of the products formed by multiplying all numbers in a row, or a column, of a determinant by their respective co-factors is the value of the determinant.

This, Laplace's, development enables us to reduce any determinant to determinants of lower order. For example, expansion of the following determinant with respect to the 4th column yields.

$$\Delta = \begin{vmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & -2 & 2 \\ 4 & -1 & 0 & 0 \\ 3 & -3 & 2 & 4 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & -1 \\ 4 & -1 & 0 \\ 3 & -3 & 2 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & -2 \\ 4 & -1 & 0 \end{vmatrix}$$

In turn, Laplace's expansion of these 3rd order determinants yields, (w.r.t. 2nd & 3rd row respectively).

$$= -2 \times 4\begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} + 2(-1)\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + 4\times4\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$$

$$(-4)(-1)\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix}$$

$$= -2\times4 \times 1 -2\times5 + 4\times4\times(-3)-(4)(-1)(1)$$

$$= -8 -10-48 + 4 = -62$$

It may be noted that the work is reduced by choosing the row or column with max. no. of zeros.

3.3. Matrices:

A matrix is a rectangular array of symbols arranged in m rows and n columns, between two double vertical lines. A square matrix has the same, number of rows and columns, or m= n. Matrices obtained from a given matrix by deleting, certain rows or columns are said to be contained in the original matrix.

The determinant of a square matrix is a determinant having the same elements as the matrix.

' Rank of a matrix '- If a matrix contains a (i.e. any) non-zero determinant of order, r, and if all determinants of order greater than r that the matrix contains have the value zero, the rank of the matrix is said to be r.

In order to utilise the algebraic approach to dimensional analysis, it is convenient to display the dimensions of the variables by a tabular arrangement called the ' dimensional matrix' of the variables. E.g. consider the following variables- V (Vel.), L,F, ρ, μ, and g.

In the following matrix, each column consists of the exponent in the dimensional expression for the corresponding variable.

	V	L	F	ρ	μ	g
M	0	0	1	1	1	0
L	1	1	1	-3	-1	1
T	-1	0	-2	0	-1	-2

The determinant formed from the last three columns in the above dimensional matrix is

$$\begin{vmatrix} 1 & 1 & 0 \\ -3 & -1 & 1 \\ 0 & -1 & -2 \end{vmatrix} = -3 \text{ or not zero}$$

Hence the order of the matrix is 3.

In general the order of the matrix is equal to the number of rows contained in the matrix. However, in certain cases, the order of the matrix is smaller than the number of rows in the matrix such matrices being called 'singular' matrices.

Now the number of dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix. Hence in general it will be (n-m), where m is the no. of fundamental dimensions, but in all cases it will be (n-r), r being the order of the dimensional matrix. In singular matrice $r < m$.

3.4. Applications of determinants and Matrices:

Determinants are used widely in the solution of systems of simultaneous equations. The following is the general rule for solving simultaneous equations-

(a) Write the coefficients of the unknowns in the order they occur as the denominator determinants.

(b) Substitute, in the determinant, the constants occurring on the right hand side of the equations for the coefficients of the unknown for which the solution is sought and let this be the numerator determinant.

(c) The quotient of these two determinants is the value of the unknown.

Example:-

$$a_1x + b_1y + c_1z = k_1$$

$$a_2x + b_2y + c_2z = k_2$$

$$a_3x + b_3y + c_3z = k_3$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \Delta_x = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}$$

$$x = \frac{\Delta_x}{\Delta}$$

$$\Delta_y = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}$$

$$y = \frac{\Delta_y}{\Delta}$$

Similarly for z.

Matrices are used in the theory of linear algebraic equations, and have many engineering applications in the field of analysis of shells, theory of turbulence etc. In the specific subject of dimensional analysis their introduction is required to state the Buckingham's theorem in a rigorous form, and to furnish its proof (not given here). Matrix notation may be used to determine the dimensionless π terms but for this purpose, its use is not obligatory.

In a general method of converting entities from one system of units to another, the theory of determinants is involved.

3.5. Arrangement of Variables:

From a given set of variables, dimensionless products can be formed in many different ways. However, in practice, some sets of products are more useful than others. The question arises - ' How may a complete set of dimensionless products be most advantageously selected, at the outset ? ' The experimenter desires that any one of the independent dimensionless variables π_1, π_2 --- be susceptible to control by experimental techniques while the others are held constant, though this may not always be possible. E.g. the velocity of fluid in a pipe can be regulated by a valve. On the other hand, the acceleration of gravity is a variable that cannot be changed.

Buckingham has pointed out that we obtain the maximum amount of experimental control over the dimensionless variables, if the original variables that can be regulated each occur in only one dimensionless product. E.g. if a velocity V is easily varied experimentally, then V should occur in only one of the independent dimensionless variables, so that it may be regulated by varying V. Similarly, if a pressure p can be easily varied, without affecting V, then p should occur in only one of the independent dimensionless variables, but not in the same one as V.

For the dependent variable, it is desired to know, how this variable depends on the other variables. Consequently, the dependent variable should not occur in more than one dimensionless product. This product may be called the ' dependent dimensionless variable'.

Since the first (m-r) variables in the dimensional matrix each occur in only one dimensionless product, the preceding conditions will be realised as nearly as possible, if the following rule is observed:

' In the dimensional matrix, let the first variable be the dependent variable. Let the second variable be that which is easiest to regulate experimentally. Let the third variable be that which is next easiest to regulate experimentally, and so on'.

(In exceptional cases, this arrangement may lead to an impasse, because the dimensional matrix does not contain a non-zero determinant of order r, in the right hand r columns. The variables in the dimensional matrix should then be rearranged, with as little alteration from the recommended rule as possible).

3.6. Examples:

Example -1: Consider again the example of drag force on a body.

$$F = f (V, D , \rho, \mu)$$

or $$f' (F, V, D, \rho, \mu) = 0$$

Since there are five variables, there will be (5-3)=2 dimensionless parameters. F is the dependent variable. Let us take μ as the other non-repeating variable leaving V, D, and ρ as repeating variables (a length, a velocity and a density- containing all the three fundamental dimensions M, L & T). Then, write the dimensional matrix in the following form:

	k_1	k_2	k_3	k_4	k_5	exponents
	F	μ	V	ρ	D	
M	1	1	0	1	0	
L	1	-1	1	-3	1	
T	-2	-1	-1	0	0	

M $\quad K_1 + K_2 + K_4 = 0$

L $\quad K_1 - K_2 + K_3 - 3K_4 + K_5 = 0$

T $\quad -2K_1 - K_2 - K_3 = 0$

Solving for k_3, k_4 and k_5 in terms of k_1 and k_2,

$$K_3 = -2K_1 - K_2$$

$$K_4 = -K_1 - K_2$$

$$K_5 =$$

$$= K_1 + K_2 + 2K_1 + K_2 - 3K_1 - 3K_2$$

$$= - 2K_1 - K_2$$

In π_1, let $K_1 = 1$, and $K_2 = 0$, and in π_2 let $K_1 = 0$ and $K_2 = 1$

<u>Matrix of Solutions</u>:

	K_1	K_2	K_3	K_4	K_5
	F	μ	V	ρ	D
π_1	1	0	-2	-1	-2
π_2	0	1	-1	-1	-1

Observe that the third, fourth and fifth columns in the matrix of solutions are merely the coefficients in the equations for k_3, k_4 and k_5 in equations expressing them in terms of k_1 and k_2. The first two columns of the matrix have all zeros except for the ' principal diagonal' which consists of 1's. This situation will always be true. Thus the matrix of solutions can be written simply from an inspection of the equations expressing the last r (usually 3) exponents in terms of the rest, i.e. (n-r) exponents.

The problem can also be solved without using the matrix notation. There are five variables - F, μ, ρ, V and D. Out of these three will be repeating variables, and the other two non-repeating. The dependent variable F should obviously be a non-repeating. According to Rouse, a length, a velocity, and a density should generally be chosen as repeating variables in Fluid Mechanics problems.

Following this precept, and giving unity exponent to the non-repeating variable.

$$\pi_1 = F^1 \quad V^a \quad \rho^b \quad D^c$$

For dimensionless term,

M $\quad 1 + b = 0$

L $\quad + 1 + a - 3b + c = 0$

T $\quad - 2 - a = 0$

$b = - 1$

$a = - 2$

$c = - 2$

Hence $\pi_1 = F^1 \quad V^{-2} \quad \rho^{-1} \quad D^{-2} = \dfrac{F}{\rho V^2 D^2}$

Again, let $\pi_2 = \mu^1 \quad V^a \quad \rho^b \quad D^c$

For dimensionless term -

M $\quad 1 + b = 0$

L $\quad -1 + a - 3b + c = 0$

T $\quad -1 - a = 0$

$b = - 1$

$a = - 1$

and $\quad c = - 1$

$$\pi_2 = \mu \quad V^{-1} \quad \rho^{-1} \quad D^{-1}$$

$$= \frac{\mu}{\rho VD} \quad \text{or} \quad \frac{\rho VD}{\mu}$$

If V were chosen as a non-repeating variable in place of μ, then F and V would be non-repeating. π_1 would be the same as before, but π_1 would be different.

$$\pi_1 = F^1 \quad \mu^a \quad \rho^b \quad D^c$$

M $\quad 1 + a + b = 0$

L $\quad 1 - a - 3b + c = 0$

T $\quad -2 - a = 0$

$a = -2$

$b = 1$

$c = 0$

$$= \frac{F \rho}{\mu^2}$$

This solution is perfectly feasible. However, note that only two independent π terms are possible. Multiply $\frac{F \rho}{\mu^2}$ by $\left(\frac{\mu}{\rho V D}\right)^2$ and we obtain the term $\frac{F}{\rho V^2 D^2}$ as before.

Example-2:

Pressure drop in a uniform pipe -

The pressure drop Δp of a liquid in a uniform pipe depends on the length L (in which the drop occurs) the diameter D, the mean velocity V, viscosity μ, mass density ρ and the average height e, of the surface roughness.

Hence $\quad f(\Delta p, \; L, \; D, \; e, \; V, \rho, \mu) = 0$

L and e have dimensions of length only. Hence the dimensionless ratios formed by them would be L/D and e/D Hence eliminating e and L from the variables, π terms have to formed from

Δp, D, V, ρ and μ

There are five variables and two π terms will be obtained. Take Δp and μ as non-repeating variables and D, V and ρ as repeating variables.

$$\pi_1 = \Delta p . V^a \quad \rho^b \quad D^c$$

M $\quad 1 + b = 0$

L $\quad -1 + a - 3b + c = 0$

T $\quad -2 - a = 0$

$b = -1$

$a = -2$

$c = 0$

Hence $\pi_1 = \frac{\Delta p}{\rho V^2}$

$\pi_2 = V^a \rho^b \; D^c$

we know that these variables form the ' .nold's no.

$\frac{VD\rho}{\mu}$ or $\frac{VD}{\nu}$

Hence $\frac{\Delta p}{\rho V^2} = f \left(\frac{VD}{\nu}, \frac{L}{D}, \frac{e}{D} \right)$

From physical considerations we can surmise that Δp per unit length of pipe will be constant, or Δp will be directly proportional to L. Also $\frac{1}{2} \rho V^2$ is the kinetic energy per unit volume and is a physically more significant parameter. Hence we may write the relationship in the special form.

$$\Delta p = \frac{1}{2} \rho V^2 . \frac{L}{D} f \left\{ \left(\frac{VD}{\nu} \right) \left(\frac{e}{D} \right) \right\}$$

The term written the brackets is the friction coefficient which is a function of the Reynolds no. and the relative roughness projection.

3.7. General Problem of Fluid Motion:

In general, the only variables that can influence fluid motion are of the following three categories-(1) the several linear dimensions fully defining the geometrical boundary conditions - a,b,c,d etc. (2) Kinematic and dynamic characteristics of flow, i.e. a mean velocity V and a pressure increment Δp (discharge Q, hydraulic gradient or boundary shear will be derivatives of these) (3) the fluid properties of density ρ, specific weight γ, viscosity μ, surface tension σ, and elastic modulus, e. Assuming that each of these characteristics is involved in the motion.

$$f'(a,b,c,d,V, \Delta p, \rho, \gamma, \mu, \sigma, e) = 0$$

The variables a, b,c and d have dimensions of length and can only form dimensionless ratios like $\frac{a}{b}, \frac{a}{c}, \frac{a}{d}$. Hence these three are written by inspection and the variables b,c, and d are eliminated from subsequent analysis.

Now, the remaining variables are -

$a, \; V, \; \Delta p, \rho, \gamma, \mu, \sigma, \; e.$

eight in number, and there will be five π terms. Let a, V and ρ be taken as repeating variables. The dimensional matrix is then arranged in the following manner -

	Δp	γ	μ	σ	e	a	V	ρ
M	1	1	1	1	1	0	0	1
L	-1	-2	-1	0	-1	1	1	-3
T	-2	-2	-1	-2	-2	0	-1	0

<u>Dimensional equations for zero dimensions of</u>

Mass $\quad k_1 + k_2 + k_3 + k_4 + k_5 + k_8 = 0$

Length $\quad -k_1 - 2k_2 - k_3 - k_5 + k_6 + k_7 - 3k_8 = 0$

Time $\quad -2k_1 - 2k_2 - k_3 - 2k_4 - 2k_5 - k_7 = 0$

Solving for k_6, k_7 and k_8 in terms of k_1 to k_5,

$$k_8 = -(k_1 + k_2 + k_3 + k_4 + k_5) \quad \text{(i)}$$

$$k_7 = -(2k_1 + 2k_2 + k_3 + 2k_4 + 2k_5) \quad \text{(ii)}$$

$$k_6 = k_2 - k_3 - k_4 \quad \text{(iii)}$$

If successive values of 1 are given to non-repeating variables in each π term, the matrix of solutions is written by inspection of equations (i), (ii) and (iii), the sixth, seventh and eighth columns being the coefficients of k_6, k_7, and k_8 respectively.

<u>MATRIX OF SOLUTIONS</u>:

	Δp	γ	μ	σ	e	a	V	ρ
π_1	1	0	0	0	0	0	-2	-1
π_2	0	1	0	0	0	+1	-2	-1
π_3	0	0	1	0	0	-1	-1	-1
π_4	0	0	0	1	0	-1	-2	-1
π_5	0	0	0	0	1	0	-2	-1

Hence $\pi_1 = \dfrac{\Delta p}{\rho V^2}$

$\pi_2 = \dfrac{\gamma a}{\rho V^2}$ or $\dfrac{V^2}{ag}$ since $\gamma = \rho g$.

$= F$

$$\pi_3 = \frac{\mu}{aV\rho} \quad \text{or} \quad \frac{Va}{\mu/\rho}$$
$$= R$$
$$\pi_4 = \frac{\sigma}{a\rho V^2} \quad \text{or} \quad \frac{aV^2}{\sigma/\rho}$$
$$= W$$
$$\pi_5 = \frac{e}{\rho V^2} = \frac{V^2}{e/\rho}$$
$$= 'C'$$

Thus we may write,

$$\frac{V^2}{(\Delta p/\rho)} = \text{Constt. } f' \left(\frac{a}{b}, \frac{a}{c}, \frac{a}{d}, F, R, W, C \right)$$

The same result could have been obtained without the matrix notation, evaluating each π term one by one.

3.8. Examples of parameters of a sediment transporting self formed channel - If we take all the variables separately, we get results involving Froude's no, Reynolds no., and variables like $\frac{\rho_s}{\rho}$, $\frac{Dm}{y_o}$ (Meandia) (depth), σ_y/y_o (coeff. of variation) etc., and there are too many terms for a useful result. Professor White reduced the number of variables by assuming that the fall velocity of grain instill water takes all the relevant solid and liquid properties into account. Then the independent variables are, discharge, sediment discharge, fall velocity, gravitational acceleration. Dependent variables are channel dimensions, Area, hydraulic mean depth, slope etc.

$$A = f\,(Q, Q_s, w, g)$$

$$\text{or} \quad f'\,(A, Q, Q_s, w, g) = 0$$

Since only two dimensions L and T are involved, there will be only two equations and the number of π terms will be three. Let A, w and Q_s be non-repeating, and Q and g repeating variables.

One π term can be written by inspection viz. $\frac{Q_s}{Q}$

Let $\pi_2 = A^1 \; Q^a \; g^b$

L $\quad 2 + 3a + b = 0$

T $\quad -a - 2b = 0$

$a = -2b$

$b = 2/5$

$a = -4/5$

$$= \frac{Ag^{2/5}}{Q^{4/5}}$$

$$\pi_3 = w^1 \quad Q^a \quad g^b$$

L $\quad 1 + 3a + b = 0$

T $\quad -1 \quad -a-2b = 0$

$$a = -\ 1/5$$

$$b = -2/5$$

$$\text{or } \pi_3 = \frac{w}{Q^{1/5} \quad g^{2/5}}$$

$$\text{Hence} \quad \frac{A g^{2/5}}{Q^{4/5}} = f\left(\frac{g^{2/5}\ Q^{1/5}}{w}\ ,\ \left(\frac{Q_a}{Q}\right)\right)$$

3.9. Velocity distribution of turbulent flow in the vicinity of a solid wall. The average, velocity U at a distance y from the boundary in straight parallel flow depends on roughness height e, a length L (say dia. in case of a pipe) the kinematic viscosity of the fluid ν , mass density of fluid ρ and boundary shear τ_o :

e and L will form the ratios $\frac{y}{e}$ and $\frac{y}{L}$

The dimensional matrix for the rest is,

	k_1 u	k_2 y	k_3 ν	k_4 ρ	k_5 τ_o
M	0	0	0	1	1
L	1	1	2	-3	-1
T	-1	0	-1	0	-2

There are five variables and two π terms are expected

M $\quad k_4 + k_5 = 0 \quad$ (i)

L $\quad k_1 + k_2 + 2k_3 - 3k_4 - k_5 \quad$ (ii)

T $\quad - k_1 - k_3 - 2k_5 = 0$

Solve for k_3, k_4 and k_5 in terms of k_1 and k_2

$$k_4 + k_5 = 0\ ,\ k_4 = - k_5$$

$$k_3 + 2k_5 = -k_1$$

$$k_3 = -2k_5 - k_1$$

Substituting for k_4 and k_3 in (ii)

$$k_1 + k_2 - 4k_5 - 2k_1 + 3k_5 - k_5 = 0$$

$$- 2k_5 - k_1 + k_2 = 0$$

$$\text{or } k_5 = -\ 1/2 k_1 + 1/2\ k_2$$

$$k_4 = +\ 1/2 k_1 - 1/2\ k_2$$

$$k_3 = -\ k_2$$

Matrix of solutions

	k_1	k_2	k_3	k_4	k_5
	u	y	ν	ρ	To
π_1	1	0	0	+1/2	-1/2
π_2	0	1	-1	-1/2	+1/2

$$\pi_1 = U \sqrt{\frac{\rho}{T_o}} \doteq \frac{u}{u_*}$$

$$\pi_2 = \frac{y}{\nu} \sqrt{\frac{T_o}{\rho}} = \frac{y u_*}{\nu}$$

$$\text{or } \frac{u}{u^*} = f\left(\frac{y\, u_*}{\nu}, \frac{y}{e}, \frac{y}{L}\right)$$

CHAPTER-IV

DESIGN OF DIFFERENT KINDS OF MODELS
CLOSED CONDUIT MODELS

4.1. Simulation of Prototype resistance:

In this kind of flow (including flow around deeply submerged bodies), gravitational and surface tension effects are absent-normally the forces involved are only viscous and inertial forces. (For unsteadly flow elastic forces also are involved).

In steady closed conduit flow Reynolds law applies. If two systems have similar boundaries of the same relative roughness, the flows will be similar in every detail when Reynolds nos. are equal. Equal Reynolds numbers are seldom possible, but almost complete similitude can be obtained by approximate equality.

The resistance law has different forms for cases of (i) laminar, (ii) smooth wall turbulent, and (iii) rough wall turbulent flows. Model procedure is different for each case.

Laminar Flow:

Prototype Reynolds no. $<$ 2000- If flow is steady and uniform inertial forces will not be involved, and complete similitude will be obtained under all conditions, so long as flow is laminar. If flow is either unsteady or non-uniform inertial forces also are involved, and the model should be designed to have the same Reynolds number as in the prototype. The time scale and other scales will apply in accordance with Reynolds Law.

Turbulent Flow:

In most of the cases prototype Reynolds number will be 10^6 or more, much beyond the transition zone between laminar and turbulent flow. In this range the relative roughness of the prototype boundary is a controlling factor, in that it determines which of the two types of surface resistance (smooth or rough) will prevail. The criteria for nature of boundary flow are as below:-

$$\frac{U_* \; k_s}{\nu} < 4 \quad \text{smooth flow}$$

$$\frac{U_* \; k_s}{\nu} \quad 4 \text{ to } 60, \text{ transition}$$

$$\frac{U_* \; k_s}{\nu} > 60 \quad \text{rough flow}$$

Complete Reynolds similitude requires not only that the resistance coefficients of model and prototype be equal, but also that the type of resistance be the same.

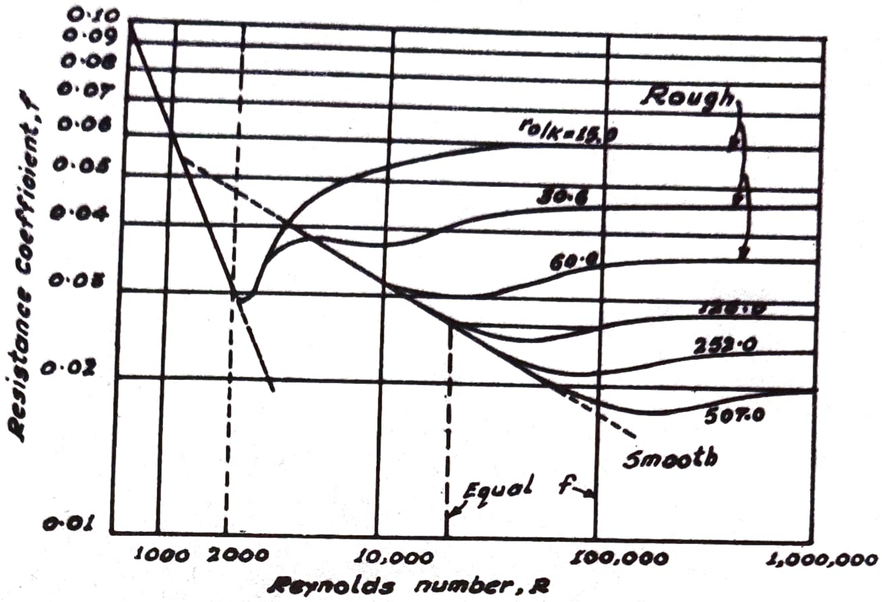

FIG. 4·1 RESISTANCE COEFFICIENTS FOR ARTIFICIALLY ROUGHENED PIPES.

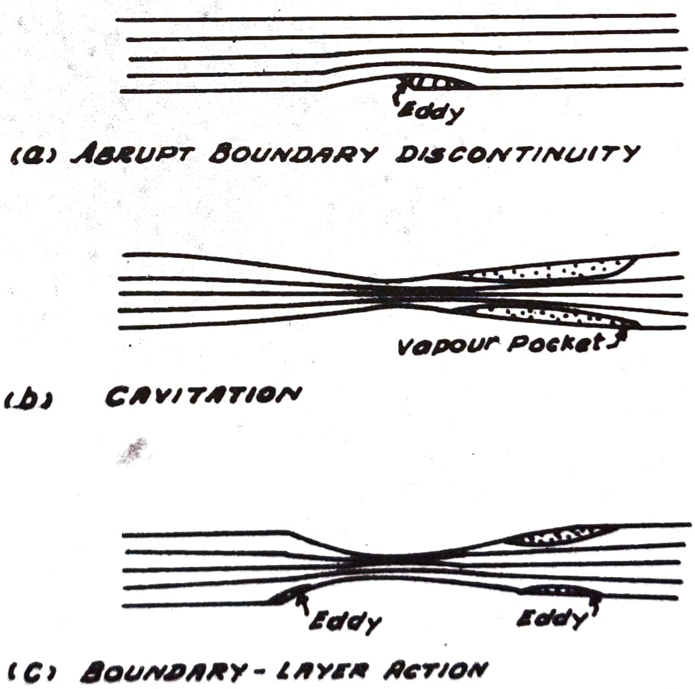

FIG. 4·2 TYPES OF SEPARATION

When the prototype boundaries are hydraulically smooth, the model boundaries are likewise made smooth, but equal resistance coefficients will be obtained only if Reynolds numbers are also the same. If the Reynolds number of the model is lower, the resistance will be disproportionately high in the model, and must be adjusted according to theory when prototype behaviour is predicted.

When the prototype boundaries are relatively rough, on the other hand, the mode of flow will generally be rough turbulent, in which there is a constant resistance coefficient equal for the same relative roughness to that of the prototype, and the results may be transferred without reservation.

When the required value of R_N cannot be obtained in model, the following expedient may be used. The resistance coefficient of the prototype can be duplicated in the model at one particular value of the Reynolds no., by making the model smoother than the prototype. For other values of flow (and hence of R_N), the resistance coefficient will be different, and the data will have to be adjusted before transference to the prototype. (Pl. see fig. 4.1.).

If R in the model can be made 10^6 or even 0.5×10^6, the errors involved are negligible for rough turbulent flow in the prototype. In most model studies inertial effects involving gates, valves, transitions, bends etc. are predominant in comparison to surface resistance. The exceptions usually involve long conduits for which difference in f results in serious discrepancies. The distorted surface losses in the conduit portion may for example, be compensated by reducing the roughness or decreasing the length. Or only the inertial loss can be determined in the model and expressed as a percentage of the total head, just above the fitting or transition.

4.2. <u>Separation Effects</u>:

Under conditions in which the flow separates from the boundaries, the geometry of the boundaries loses its primary significance, and similitude is largely dependent upon the dynamic action of the fluid. Separation may occur when (1) there is abrupt change or discontinuity of a boundary (2) velocities exceed those required for incipient cavitation, and (3) the boundary layer expands in the face of an adverse pressure gradient (Pl. see Fig.4.2.)

In the first case the point of a separation is fixed and will be the same in the model and the prototype and the eddy formation will be fairly similar to that in the prototype, if the Reynolds numbers are of the same order of magnitude.

Separation due to cavitation is governed by the criteria separately given for cavitation.

Conditions which determine the location of boundary layer separation are complex, for they include the geometry and roughness of the boundary, and the Reynolds number and turbulence of the flow. Moreover, a pronounced shift in separation point often occurs when the boundary layer changes in type from laminar to turbulent. If a model is run at velocities high enough to ensure a turbulent boundary layer, the distortion in the location of the point of separation will be small. Definite assurance of similarity in the point of separation can be obtained only if the relative roughness, as well as the Reynolds no., are the same in the model and the prototype.

4.3. Cavitation Studies:

Cavitation in Hydraulic structures results when the pressure falls below the vapour pressure of the liquid at some point in the system. Its occurrence may cause (a) damage by pitting (b) serious vibration and (c) marked reduction in flow efficiency. The particular effect of cavitation being studied determines the type of model test. In general, there are two techniques (1) Operation of model at atmospheric pressure.

(2). Operation at Scaled atmosphere, or reduced pressure:

When the problem involves the prediction or elimination of cavitation erosion, the first technique is suitable. Detailed measurements of pressure distribution in the model will reveal sub-atmospheric pressures which, scaled to prototype terms, may approach the vapour pressure (E.g. a negative head of 1.5 m. in model will correspond to 9 m. in the prototype if length ratio, L_r is 6). This should be regarded as evidence of possible cavitation in the prototype. Negative pressures exceeding one half the atmospheric pressure ar considered intolerable in the prototype, as local irregularities may cause local cavitation. To detect negative pressure areas, piezometer connections should be very carefully located at all possible locations.

The vibration and energy loss effects of cavitation can be studied best in reduced pressure models, such that the pressure at the test section can be maintained at any desired degree of vacuum. Such a test facility makes it possible to duplicate cavitation conditions in the model by arranging equal values of the 'cavitation number' in model and prototype. Cavitation number is written as,

$$\sigma = \frac{h_o - h_v}{V_o^2/2g}$$

where h_o is the piezometric head at some reference section, h_v is the vapour pressure head of the model fluid at ambient temperature and $V_o^2/2g$ is the reference velocity head.

For the same value of σ in model and prototype, the pattern of cavitation, and hence the flow efficiency, will be the same. Since the frequency of vibration and the rate of pitting will vary in proportion to the velocity however, and since the physical properties of the boundary material will also be involved, such factors must be given proper condition in predicting the structural behaviour of the prototype from model tests.

Example-A baffle pier is to be tested in a free-surface water tunnel with a length ratio $L_r = 1/25$. What pressure should be maintained inside the tunnel to make the cavitation index the same for model and prototype ? For the prototype atmospheric pressure is 10.3 m and vapour presssure 12.5 cms. of water at 10°C.

The following cavitation condition has to be satisfied -

$$\frac{(h_o - h_v)_m}{(V_o^2 / 2g)_m} = \frac{(h_o - h_v)_p}{(V_o^2 / 2g)_p}$$

or

$$\frac{(h_o - h_v)_m}{(h_o - h_v)_p} = \frac{(V_o^2 / 2g)_m}{(V_o^2 / 2g)_p}$$

Now since it is a free surface problem, Froude's criterion must also be satisified. From Froude's Criterion $V_r = \sqrt{L_r}$

and

$$\frac{(V^2/2g)_m}{(V_o^2/2g)_p} = L_r$$

Hence

$$\frac{(h_o - h_v)_m}{(h_o - h_v)_p} = L_r = 1/25$$

or

$$h_{om} = 1/25 \ (h_o - h_v)_p + h_{vm}$$

$$= 1/25 \ (10.3 - 0.125) + 0.125$$

$$= 0.532 \text{ m abs.}$$

At this pressure, the model will indicate the location and extent of cavitation in the prototype if the Froude numbers are the same.

4.4. <u>Compressibility in Unsteady Flow</u>:

Problems in which compressibility is important are seldom encountered in hydraulics except in cases of unsteady flow. E.g. the design and operation of closed conduit systems, such as pumping plant delivery lines, power stations, and surge tanks, involve knowledge of the elastic behaviour of liquid and conduit under transient conditions, and detailed model studies of such problems must, hence, be based upon the Mach Law of similitude. However, most of these problems can be handled adequately by analytical or graphical methods, and model studies are not always necessary.

A method developed by Gibson (The investigation of the surge tank problem by model experiments - Proc. Inst. C.E. Vol.219 Pt.I, 1924-25, p.161, abstract Guthrie Brown Vol.I, p.127) for model study of surge tanks will be dealt with later.

CHAPTER-V

OPEN CHANNEL MODELS

5.1. General:

The technique of open channel models is complicated by the presence of an open water surface, which is free to change in position and shape. Usually the shape of the surface in the prototype is not precisely known, though we do know that it is a surface of constant pressure comprising one of the flow boundaries. The dominating force is that of gravity or weight, and thus the Froude Law of similitude is pertinent, with some adjustments and limitations, which often require scale distortions to accomodate the effect of other forces.

Problems of free surface flow involving hydraulic structures generally consist of three parts - (1) overflow and underflow sections (2) transitions, and (3) energy dissipators, any or all of which may be included in a single model. It is customary to preserve complete geometric similarity, and model heads are adjusted to the values required by the Froude's law. The testing technique, as well as the interpretation of model results, varies somewhat for the different ports of a structure, each of which will be discussed separately.

Problems of canal or river hydraulics, on the other hand, involve the prediction of stage discharge relationships, scour and deposition, and the passage of floods, and can seldom be studied with undistorted models. Although the Froude Law is still the basic similitude criterion, surface roughness and viscous resistance must be given careful consideration. Problems of river models will be considered in detail later.

5.2. Overflow and Underflow Structures:

Examples of these structures are spillways and out let works, diversion dams, sluiceways, canal intakes, waste weirs, chutes, regulators and falls. All these structures usually involve change of flow from subcritical to hyper-critical. In India the model boundaries generally consist of smooth concrete or cement plaster, and occasionally of planed wood or sheet metal- all these surfaces provide the required smoothness. With scale ratios of 1/30 to 1/60, the effects of distorted resistance in the model are small in comparison to the inertial effects and results are transferable with few reservations. More precise calibrations may, however, be obtained with enlarged sectional models. Such models which may have a scale ratio of 1:5 to 1:10 should be at least 2 ft. in width, and approach conditions should be arranged with care. The best shape of overflow profile, as well as the profile of the water surface, may also be obtained with a higher degree of accuracy in such a model.

When the problem involves overfall shapes with high coefficients and sub-atmospheric pressures, the existence and effects of adverse pressure gradients likely to cause separation should be investigated. The procedure for this purpose follows that discussed earlier.

In addition to physical measurements, the general appearance of flow should be carefully noted, as it will often indicate a need for revision not apparent in physical data. Often small disturbances in the model have intolerable adverse effects in the prototype, and a judgement on these is based on experience rather than on theory.

Example - spillway-prototype crest length 500 ft., max. discharge 200,000 cfs, C in prototype approximately 3.80.

To find L_r, Q_m and H_m available lab. discharge being 5 cusecs.

$$H_p = \left(\frac{200,000}{3.80 \times 500} \right)^{2/3} = 22.3 \text{ ft.}$$

If full capacity of lab. is utilised $Q_m = 5$ cusecs.

$$L_r = \left(\frac{5}{200,000} \right)^{2/5} = \frac{1}{69.5} \text{ nearly}$$

Let the scale be taken as 1/70

$$\text{Then } Q_m = Q_p \; L_r^{5/2} = \frac{200,000}{41,000} = 4.88 \text{ cusecs.}$$

$$H_m = \frac{22.3}{70} = 0.319 \text{ ft.}$$ which is more than the minimum of 0.25 ft. considered necessary to eliminate other forces. Hence satisfactory.

5.3. Transition Structures:

Open channel transition structures may involve changes in elevation or changes in plan. Model tests on transition structures are very desirable, specially in super critical flow.

The model data on water surface profiles and pressure distribution may generally be transferred to the prototype by the Froude relationships alone. However, when unusually large divergences are involved, the influence of the distorted boundary layer on separation should investigated by methods previously discussed. In long flat transitions, boundary resistance in the model may be disproportionately high, and a distortion in slope or length may be required to offset it. The distortion can be accomplished on the basis of computed losses in model and prototype, as explained later in conjunction with fixed bed models.

When the prototype velocities are high, as in case of long steep chute or spillway, the sheet of water entrains appreciable percentages of air, thereby increasing the depth of flow. This phenomenon which is known as insufflation is not duplicated in the ordinary model based on Froude Law of similitude. It is, therefore, necessary to make allowance for bulking when determining heights of training walls from model studies. Special models wherein high velocities are initiated by discharge from a pressure tank into the model chute may be used to accomplish insufflation for qualitative studies.

5.4. Energy Dissipators:

The energy dissipator is designed to protect the main structure from damage by fast flowing water - the energy dissipator is itself a major structure and should not be washed away. The conventional method of evaluating stilling-basin performance is by the general apperance of flow, especially for preliminary designs. The presence of extreme turbulence, waves, high exit velocities, and high velocity eddies or pulsating flow in the downstream portion of the structure indicates the need for refinement of design or abandonment of the particular plan. Each design should be tested over the complete range of discharge and tail water conditions. Basins that do not have a fairly wide range of permissible tail water are ordinarily not practical, since many unknown variables such as degradation of stream beds or power plant discharge are not reflected in the preparation of the tail water curve. Refinment in designs of stilling basins may be evaluated by comparing measured (1) velocities (2) wave heights, (3) pressures and (4) erosion of the downstream river bed. The general operating conditions, preferably recorded on photographs will also help to evaluate the effectiveness of a particular design.

Wave and surge heights are particularly important where the discharge is passed directly from the stilling basin into an unlined canal since the waves have a destructive effect on the sloping banks. Maintenance costs can be reduced considerably if waves and surges can be held to a minimum. In such cases it is desirableto record wave heights and period by motion pictures. Oscillograph records or other instantaneous methods.

The most objective criterion to judge stilling basin behaviour is that of comparative erosion. To make qualitative erosion studies, a readily erodible sand or gravel is used to represent the channel downstream from the stilling basin. The measured depth and extent of erosion indicate the relative effectiveness of the basin under test when compared with erosion patterns from other basins if the same bed material is used. The erosion pattern and depth in the model may or may not represent that in the prototype-depending on the model material. If the model material just starts to erode at model velocity corresponding to prototype velocity just competent to start erosion in the prototype, scour measurements will be quantitatively applicable, at least approximately. The duration of test is not based on Froude's criterion. The flow should be maintained till measurable (and fairly stable) scour develops, and the duration should be held constant in all comparative runs.

5.5. **Fixed bed channels:**

Problems involving relatively long stretches of either a canal or a river, wherein actual changes in bed configuration are not critical are usually studied in fixed-bed models. The influence of wall resistance is generally of major importance in such problems and cannot be ignored or adjusted when the results are interpreted.

Unless a very large model is made, distortion in slope is required (1) to offset the disproportionately high resistance (2) to ensure sufficiently high. Reynolds no. for turbulent flow, or (3) to accommodate the model in the available space. In the first case, the vertical exaggeration is designed to compensate for high model resistance. In other cases extra or artificial model roughness may be required to compensate for exaggerated model slope.

The required distortion of slope can be computed with sufficient precision from the emperical Manning formula -

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \quad \text{(foot units)}$$

In order that the velocity ratio will vary with the square root of the depth scale, as required by Froude's Law -

$$V_r = \sqrt{y_r} = \frac{R_r^{2/3} S_r^{1/2}}{n_r}$$

If the model were undistorted $S_r = 1$, $R_r = y_r = L_r$,

Then , $V_r = L_r^{2/3} / n_r = \sqrt{L_r}$

or $n_r = L_r^{1/6}$ (1)

Since we know that n varies as 1/6th power of the roughness projection K_s, it means that K_s should be reduced in the same ratio as the length scale- this is often impractical and even if practical may change the nature of the boundary layer itself, from rough turbulent to smooth turbulent- in extreme cases the flow may become laminar (in movable bed models similiarity of sediment movement imposes its own condition). Substituting y_r/L_r for S_r results in the expression,

$$\sqrt{y_r} = \frac{R_r^{2/3} y_r^{1/2}}{L_r^{1/2} n_r}$$

or

$$\frac{y_r}{L_r} = \frac{n_r^2 y_r}{R_r^{4/3}} \quad (2)$$

Also $Q_r = L_r \cdot y_r \cdot V_r = L_r (y_r)^{3/2}$ (3)

<u>For wide channels</u>:

$$R_r = y_r, \quad \text{and}$$

$$\frac{y_r}{L_r} = \frac{n_r^2}{y_r^{1/3}}$$

or $L_r n_r^2 = y_r^{4/3}$ (4)

or $y_r = L_r^{3/4} n_r^{3/2}$ (5)

If, $n_r = 1$,

$y_r = L_r^{3/4}$ (6)

Also

$$L_r y_r^{3/2} = Q_r$$

and

$$L_r = \frac{y_r^{4/3}}{n_r^2}$$

Hence

$$y_r^{(4/3 + 3/2)} = Q_r \quad n_r^2$$

or, $y_r^{17/6} = Q_r n_r^2$ (7)

If n is known for the model and prototype, n_r is also known, and the exaggeration y_r/L_r can be adjusted for a given depth y and hydraulic radius R. In models for which the slope distortion is dictated by other considerations, an adjustment of model roughness is required to duplicate the prototype conditions. If the distortion, and the value of n for the prototype are known, the required value of n for the model can be computed from the equation 5. Unless basic data are available in a particular laboratory on the values of n for different types of roughness, the required roughness adjusted for a particular depth, will yield dependable results for flow at or near that depth. If a problem involves several depths, the model roughness should be adjusted to give an average friction that is approximately right for each depth, or the roughness may be varied with depth for a closer approximation at all depths.

It may be noted that Manning's formula applies only at sufficiently high values of R for viscous effect to be negligible.

5.6. <u>Scale for Uniform Channels</u>:

If channel under study in uniform, Froude's law need not necessarily be satisfied. Any properly corresponding set of scales of linear dimensions, discharge and velocity, time and roughness will ensure that the model will reproduce prototype surface profiles. These relations are prototype

$$V_p = \frac{1.49}{n_p} R_p^{2/3} S_p^{1/2}$$

for the model $V_m = \frac{1.49}{n_m} R_m^{2/3} S_m^{1/2}$

Hence $V_r = \frac{V_m}{V_p} = \frac{R_r^{2/3} S_r^{1/2}}{n_r}$

Since $S_r = \frac{y_r}{L_r}$

$$V_r = \left\{ \frac{R^{2/3} y^{1/2}}{n L^{1/2}} \right\}_r \qquad (4)$$

$$Q_r = (L_r y_r V_r)$$

$$= \left\{ \frac{R^{2/3} L^{1/2} y^{3/2}}{n} \right\}_r \qquad (5)$$

The time scale can be derived from the following relations,

$$T_r = \frac{\text{Volume scale}}{\text{discharge scale}} = \left(\frac{L^2 y}{Q} \right)_r \qquad (6)$$

In non-uniform flow there is interchange of velocity and pressure heads from section to section, and for correct reproduction of water surface profiles it is necessary that Froudian Law should be satisfied. However, in actual practice small departures from Froude's Law are tolerated as they lead to negligible errors.

Example to illustrate the use of Manning's criterion for rigid bed channels -

Consider a rectangular prototype channel 90 m wide by 3 m deep.

First - Undistorted model

$$n_r = L_r^{1/6}$$

If L_r is fixed, n_r must satisfy the above relationship. E.g. if $L_r = \frac{1}{60}$, $n_r = \left(\frac{1}{60}\right)^{1/6} = 0.505$

If the prototype channel has a concrete surface with $n_p = 0.014$, the required model $n_m = 0.0071$. It would be impractical to get this value of n, and even if obtained will change the nature of boundary flow from rough to smooth.

Second:- Distorted model, L_r and y_r fixed for other considerations, to find n_r

Let $L_r = \frac{1}{60}$, $y_r = 1/30$

Model width = 1.5 m and depth = 0.1 m

$$R_p = \frac{90 \times 3}{96} = 2.81 \text{ m}$$

$$R_m = \frac{1.5 \times 0.1}{1.5 + 0.2} = 0.0884 \text{ m}$$

$$\therefore R_r = \frac{0.0884}{2.81} = \frac{1}{31.8}$$

$$n_r^2 = \frac{y_r}{L_r} \left(\frac{R_r}{y_r}\right)^{4/3} = \frac{R_r^{4/3}}{L_r}$$

$$= 60 \left(1/31.8 \right)^{4/3} = 0.60$$

$$n_r = 0.775$$

This is a more practical value. The Reynolds number for total flow, as well as nature of boundary flow are more conducive to similarity.

Third: Let n_r be fixed - we may have to use the same type of surface as in the prototype, so that $n_r = 1$

Also let L_r be $\frac{1}{60}$ as before, to find y_r

$$R_r^{4/3} = n_r^2 \; L_r = 1. \frac{1}{60} = \frac{1}{60}$$

$$R_r = \left(\frac{1}{60} \right)^{3/4} = \frac{1}{21.5}$$

Now, $R_p = 2.81$ m

Hence $R_m = \frac{2.81}{21.5} = 0.1305$ m

$$\frac{1.5\, y_m}{1.5 + y_m} = 0.1305$$

From which, $y_m = 0.16$ m

And, $y_r = \frac{0.16}{3} = \frac{1}{18.8}$

Distortion, $\frac{y_r}{L_r} = \frac{60}{18.8} = 3.19$

In each case $Q_r = L_r \quad y_r^{3/2}$

If Q_r is fixed, then out of L_r, y_r and n_r only one can be chosen.

CHAPTER-VI

MOVABLE - BED CHANNELS

6.1. Introduction:

There are many open-channel problems, notably river problems, involving movement of sediment. Such problems are usually studied in movable bed-models, though limited studies for current directions and velocities can also be made in fixed bed models. Only limited similitude can be attained in movable bed models, which are practically always distorted vertically. Even then they have proved their worth in giving dependable answers to river problems.

The application of mathematical approach to similitude in movable bed models is very difficult. The number of equations involved is large and some of these e.g. the bed load equation and the flow equation for mobile bed channels are themseleves only approximately defined. An effort at deriving scale relationships from known equations for distorted, movable bed models made by Einstein and Ning Chien (TASCE Vol.121, 1956' Similarity of Distorted River Models with Movamble Beds') will be discussed later.

The common procedure is to select horizontal scale on basis of available space or discharge. The distortion in the vertical scale is taken on basis of earlier experience on successful river models of similar characteristics. Distortion of more than six is usually considered inadvisable, though some successful models have been made and run with larger exaggeration. Distortion of four or less is considered desirable. Greater distortion is permissible in wide shallow streams than in narrow deep ones.

Distortion becomes necessary to ensure sufficient movement of model bed material, yet it introduces certain undesirable effects - the exaggeration may increase the slopes of the model banks beyond their angle of repose so that they will not longer stand; if the banks can be assumed non erodible they can be made of a rigid material. Distortion also increases the longitudinal slope of the stream and tends to increase velocity and reduce water depth to such an extent that artificial model roughness may be required to restore it. The bed roughness is a function of the bed material which is amenable to limited adjustment only. If some natural materials are used so that the sp. gravity ratio is one, adequate movement requires smaller material while roughness needs a larger size. By using light materials, like coal (sp.gr.1.30) pumice (1.7), resins (1.09 to 1.13), etc. better results can be obtained in this respect. Even so, there may be distortion in discharge and velocity scales from the theoretical Froudian Law . If such departure is large, water surface predictions of the model may not be very reliable.

Another effect of vertical exaggeration is distortion of lateral distribution of velocity. Hence in problems of confluence of two streams, channels split by islands and in case of sharp beds, the distortion should be kept as small as possible.

6.2. Verification:

A necessary feature of movable bed models is 'verification' i.e. obtaining scour and deposit effects in the model corresponding to known past scours and deposits in the prototype by running prototype discharges adjusted to trial scales of discharge and time. If the verification is successful, then these trial scales can be adopted for further studies. The essential requisite for verification is an accurate survey of the prototype, depicting events that are to be verified (e.g. before and after a flood) and hydro logical data for the events.

The verification process must be limited to certain prescribed events:-

(a) The known event in the prototype for model verification should involve phenomenon pertinent to the proposed study. E.g. reproduction of flood gauges is not adequate for a model meant to study bed movement.

(b) The event in the prototype must represent a continuous action of reasonable duration. Thus a model meant to study river stability should not be verified for a single high flood but for at least an entire flood season.

(c) The event in the prototype should represent normal, yearly, phenomena. Freak floods resulting from hurricanes should be avoided.

(d) The more the plans to be tested in the model depart from conditions under which the model was verified, the less trust worthy become the resulting data. Thus data from the test of a system of dykes which change the regimen of the river only slightly are to be held more trust worthy than data from a series of cut offs which markedly change the regimen of the river.

The actual verification of a movable bed model resolves itself into a trial and error process of adjustment. The model is set up to accord with conditions as they existed in the prototype immediately prior to the beginning of the verification event. It is then a case of determining that set of operating conditions which will result in reproduction of the event. Among the items that may be adjusted are- discharge scale, water surface slopes, types of bed materials, magnitude of time scale, roughness of fixed boundaries, rate of feeding of bed materials. Sometimes a variable time scale has to be adopted for different stages of flow.

6.3. Einstein and Ning Chien's approach to design of River models with movable beds (T.A.S.C.E. Vol.121,1956, p. 440).

6.3.1. Distortions:

In this study by the authors several distoritions are contemplated:

1. If the ratio of horizontal lengths L_r is independent of depth ratio h_r, the model is vertically distorted.

2. If the grain size ratio, D_r, is different from L_r and h_r a third length ratio is introduced and with it a second distortion.

3. If the slope ratio S_r is chosen independent of L_r and h_r, the model is assumed to be tilted in addition to other distortions.

4. If the ratio of effective densities of sediment $(\rho_s - \rho_f)_r$ is assumed to be different from the ratio of fluid densities ρ_{fr} (which is unity) there is a fourth distortion.

5. A fifth distortion is introduced in the time scale if the duration of flood stages in the model is taken to a different time scale than the one obtained from velocity and sediment movement considerations.

6. A sixth distortion is a result of the impossibility of obtaining suspended - load rates in a model in the same scale at which the bed load rates are reproduced, i.e. the bed load rate ratio is different from the total load rate ratio

7. A seventh and last distortion permits the ratio of settling velocities V_{sr} of corresponding grains to be differen from the ratio of the corresponding flow velocities.

6.4. Relationships Describing Alluvial Flows:

(1) Friction criterion:- Assume a flow equation of the form,

$$V = \frac{C\sqrt{g}}{D^m} \quad S^{1/2} \quad h^{1/2+m}$$

In the Manning form m = 1/6. However in the general case the resistance law may be different from Mannings and should be determined by trial. Assuming that the exponent m is the same for the model and the prototype, we get the ratio relationship -

$$\frac{V_r^2 \quad D_r^{2m}}{S_r \; h_r^{1+2m} \; C_r^2} = \Delta v \qquad (1)$$

For exact similarity $\Delta v = 1$

(2) Froude Criterion:

This gives,

$$\frac{V_r}{h_r^{1/2}} = \Delta F \qquad (2)$$

Again for exact similarity $\Delta F = 1$

(3) & (4) **Sediment Transport Criterion:**

According to Einstein this relationship is defined as ϕ_* is a function of ψ_*. Neglecting the effect of grading.

$$\phi = \frac{q_B}{g(\rho_s - \rho_f)} \left(\frac{\rho_f}{\rho_s - \rho_f}\right)^{1/2} \left(\frac{1}{gD^3}\right)^{1/2}$$

q_B is the bed load transport by weight/unit width/unit time.

If ϕ is to be equal in model and prototype, and if the same liquid is used so that $\rho_{fr} = 1$,

It follows that

$$\frac{q_{Br}}{(\rho_s - \rho_f)_r^{3/2} \, D_r^{3/2}} = 1 \qquad (3)$$

Again, neglecting the effect of grading,

$$\psi = \frac{\rho_s - \rho_f}{\rho_f} \cdot \frac{D}{R'_b \, S}$$

Where R'_b is the hydraulic mean depth of surface drag only (neglecting form drag). Let $R'_b = \eta h$ where h is the total depth, corrected for side friction where necessary.

Then ψ too should be equal in the model and the prototype

$$\frac{(\rho_s - \rho_f)_r \, D_r}{\eta_r \, h_r \, S_r} = 1 \qquad (4)$$

(5) **Laminar Sublayer Criterion:**

For similarity the relative projection of the grain out side the laminar sub-layer should be the same in the model and the prototype i.e. $\delta_r = D_r$

Now δ is proportional to $\frac{\nu}{U_*}$ Assuming ν to be equal in model and prototype,

$\delta \propto \frac{1}{U_*}$, U_* being the shear velocity.

i.e. $\delta \propto \frac{1}{\sqrt{R'_b g S}}$

Hence $\delta_r = \frac{1}{\eta_r^{1/2} h_r^{1/2} S_r^{1/2}}$ and equals D_r

$$(5)$$

or $D_r \, \eta_r^{1/2} \, h_r^{1/2} \, S_r^{1/2} = \Delta\delta$

$\Delta\delta$ should be unity but a slight deviation is permissible.

(6) The ratio of bed load to total load in model and prototype should be determined,

$$\text{Let} \quad \frac{q_{Br}}{q_{Tr}} = \frac{1}{B} \qquad (6)$$

(7) Hydraulic time ratio is defined by the equation,

$$\frac{L_r}{V_r} = t_{1r} \qquad (7)$$

(8) Time t_{2r} is the time scale at which the prototype hydrographs must be repeated in the model, thus it indicates the duration of individual flows. Ratios of this time must be such that corresponding time intervals are required by corresponding sediment rates q_T to fill corresponding volumes. Expressed in ratios, this equation can be written for the unit width as-

$$q_{Tr} \times t_{2r} = L_r \, h_r \, (\rho_s - \rho_f)_r \qquad (8)$$

q_T is measured as submerged wt./unit time/ unit width.

(9) Tilt,

$$\frac{S_r}{h_r/L_r} = \Delta N \qquad (9)$$

Where ΔN is the additional slope provided by tilt. Such tilt is not permissible in reversible or tidal flows.

The nine independent equations given above are tabulated in table No.6.1.

As will be seen from the table there are 18 variables in all as below:-

13 ratios (first thirteen columns)

4 Δ- values (last four columns)

1 - exponent in the flow equation, m.

Out of these the four Δ values, m and three of the ratios B, C_r and η_r are assumed known or computed from known data. This leaves ten unknown ratios and nine equations. Thus free choice can be exercised for only one ratio, and all others will be determined from it (if no deviation in values is to be allowed). For L_r fixed independently, solution for other ratios is given in Table 6.2. Solutions have also been given by the authors for h_r and $(\rho_s - \rho_f)_r$ chosen independently.

tabulated in table No.6.1.

for other ratios is given in Table 6.2.Solutions have also bee

TABLE 6:1

EXPONENTS FOR MODEL LAWS FOR RIVER MODELS WITH SEDIMENT MOTION
(All quantities transposed to left hand side, right hand side = 1)

Eq.	L_r	h_r	V_r	S_r	D_r	$(\rho_s-\rho)_r$	q_{B_r}	q_{T_r}	t_{1r}	t_{2r}	B	C_r	η_r	ΔV	ΔF	$\Delta\delta$	ΔM	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1		$-1-2m$	2	-1	2m							-2		-1				=1
6		-1	2												-2			=1
8					-3	-3	2											=1
10		-1		-1	1	1							-1					=1
11		1		1	2								+1			-2		=1
13							1	-1			1							=1
14	-1		1															=1
15	-1	-1				-1		1		1								=1
16	-1	-1		1													-1	=1

a- The values of B, C_r, η_r and m are determined from auxiliary computations. Values of Δ may be chosen to suit the conditions.

TABLE: 6-2

MODEL RATIOS FOR OPEN CHANNEL FLOWS WITH SEDIMENT MOTION
(b) LENGTH RATIO, L_r CHOSEN

Symbol	h_r	L_r	$(\rho_s - \rho_f)_r$	C_r	η_r	B	ΔF	$\Delta \delta$	ΔN	ΔV
h_r	..	$\frac{m+1}{4m+1}$	..	$\frac{-2}{4m+1}$	$\frac{-m}{4m+1}$	..	$\frac{2}{4m+1}$	$\frac{2m}{4m+1}$	$\frac{-(m+1)}{4m+1}$	$\frac{-1}{4m+1}$
V_r	..	$\frac{m+1}{2(4m+1)}$	..	$\frac{-1}{4m+1}$	$\frac{-m}{2(4m+1)}$	..	$\frac{2(2m+1)}{4m+1}$	$\frac{m}{4m+1}$	$\frac{-(m+1)}{2(4m+1)}$	$\frac{-1}{2(4m+1)}$
S_r	..	$\frac{-3m}{4m+1}$	..	$\frac{-2}{4m+1}$	$\frac{-m}{4m+1}$	..	$\frac{2}{4m+1}$	$\frac{2m}{4m+1}$	$\frac{3m}{4m+1}$	$\frac{-1}{4m+1}$
D_r	..	$\frac{2m-1}{2(4m+1)}$	..	$\frac{2}{4m+1}$	$\frac{-(2m+1)}{2(4m+1)}$	..	$\frac{-2}{4m+1}$	$\frac{2m+1}{4m+1}$	$\frac{1-2m}{2(4m+1)}$	$\frac{1}{4m+1}$
$(\rho_s - \rho_f)_r$	..	$\frac{3(1-2m)}{2(4m+1)}$	..	$\frac{-6}{4m+1}$	$\frac{3(2m+1)}{2(4m+1)}$	..	$\frac{6}{4m+1}$	$\frac{2m-1}{4m+1}$	$\frac{3(2m-1)}{2(4m+1)}$	$\frac{-3}{4m+1}$
q_{Br}	..	$\frac{3(1-2m)}{2(4m+1)}$	..	$\frac{-6}{4m+1}$	$\frac{3(2m+1)}{2(4m+1)}$	..	$\frac{6}{4m+1}$	$\frac{6m}{4m+1}$	$\frac{3(2m-1)}{2(4m+1)}$	$\frac{-3}{4m+1}$
q_{Tr}	..	$\frac{3(1-2m)}{2(4m+1)}$	..	$\frac{-6}{4m+1}$	$\frac{3(2m+1)}{2(4m+1)}$	1	$\frac{6}{4m+1}$	$\frac{6m}{4m+1}$	$\frac{3(2m-1)}{2(4m+1)}$	$\frac{-3}{4m+1}$
t_{1r}	..	$\frac{2m+1}{2(4m+1)}$	..	$\frac{1}{4m+1}$	$\frac{m}{2(4m+1)}$	..	$\frac{-2(2m+1)}{4m+1}$	$\frac{-m}{4m+1}$	$\frac{m+1}{2(4m+1)}$	$\frac{1}{2(4m+1)}$
t_{2r}	..	$\frac{5m+2}{4m+1}$	..	$\frac{-2}{4m+1}$	$\frac{-m}{4m+1}$	-1	$\frac{2}{4m+1}$	$\frac{-(2m+1)}{4m+1}$	$\frac{-(m+1)}{4m+1}$	$\frac{-1}{(4m+1)}$

6.5. Example: A river in flood carries 400,000 cusecs. The resistance law applicable to the river as well as to model is $\frac{V}{V_*} = 8.1 \left\{ \frac{R}{k_s} \right\}^{1/6}$

If lab. space permits a length of scale of 200, determine the other scales.

Here $m = 1/6$ (Ratios are prototype/model)

$C = 8.1$

Assume that $\eta = 1$ and all $\measuredangle$ values are also 1

Then $h_r = L_r^{\frac{m+1}{4m+1}} \; C_r^{\frac{-2}{4m+1}}$

$= L_r^{0.7} = 41$ (exaggeration= $\frac{200}{41}$ = nearly 5 times)

$$D_r = L_r^{(2m-1)/2(4m+1)}$$

$$= (200)^{-1/5} = \frac{1}{2.88} \text{ or } 0.348$$

This means that model material should be 2.88 times larger than prototype material.

$(\rho_s - \rho_f)_r = L_r^{0.6}$, or

$$(\rho_s - \rho_f)_r = D_r^{-3} = (0.348)^{-3}$$

$$= 23.9$$

If the submerged density in prototype is (2.65-1.00) = 1.65, that in the prototype should be 0.069. The model bed needs material of density 1.069 gms/c.c.

6.6. Einstein-Ning Chien approach indicates that similarity in a mobile bed channel can not be truely obtained without use of light weight material. This fact can be demonstrated in a simpler way also. Only two conditions are chosen, first Manning- Froude which gives

$$L_r \, n_r^2 = y_r^{4/3} \qquad (10)$$

With respect to sediment movement, the only condition imposed is that the ratio of bed stress to critical stress is the same in the model and the prototype. Or

$$\frac{\tau_{or}}{\tau_{cr}} = 1 \qquad (11)$$

Or

$$\frac{\gamma_{wr} \, y_r \, S_r}{(\gamma - \gamma_w)_r \, D_r} = 1$$

Since $\gamma_{w\,r}$ is unity, and $S_r = \frac{y_r}{L_r}$, this leads to

$$y_r^{\,2} = L_r\,(\gamma - \gamma_w)_r\;\;D_r \qquad (12)$$

Assuming, $n_r = D_r^{\,1/6}$ which will be true where bed ripples are not important equation (10) becomes,

$$L_r\,D_r^{\,1/3} = y_r^{\,4/3} \qquad (13)$$

Between equation (12) and (13) there are four unknown ratios, viz. L_r, y_r, D_r and $(\gamma - \gamma_w)_r$. Hence, theoretically, any two can be freely chosen, and the others determined. But if $(\gamma - \gamma_w)_r$ is taken equal to unity in equation (12), the solution obtained is $L_r = y_r = D_r$, or an undistorted model. This solution is feasible because no condition has been imposed to ensure the same nature of main flow and boundary flow as in the prototype. As soon as these necessary checks are made, this solution will have to be ruled out.

Let us take L_r the same as in the Einstein-example. This may be fixed by limitations of space.

Then assuming various values of y_r, D_r and $(\gamma - \gamma_w)_r$ are calculated and tabulated as below:-

L_r	y_r	D_r	$(\gamma - \gamma_w)_r$	Remarks
200	25	0.049	64	All ratios are prototype to model
	40	0.32	25	
	50	0.78	16	
	100	12.50	4	

It will be seen that for $y_r = 41$, the result will be practically the same as by Einstein- Ning Chien. Since no condition for nature of flow has been imposed, this will have to be exercised separately.

6.7. Alen (Scale models in Hydraulic Engineering) has given the following argument in respect of settling velocity of silt in distorted models.

'If the horizontal scale is 1: x and vertical scale 1:y the velocity scale for horizontal movement is $1 : \sqrt{y}$, and time scale $1 : x / \sqrt{y}$. But since all vertical depths are exaggerated in the ratio of x:y, it follows that a suspended particle should have a vertical motion $x/y \cdot 1/\sqrt{y}$ or $x/y^{\,3/2}$ times as fast as a corresponding particle in nature.

Lacey has extended this argument to state that the scales should be so chosen that the vertical motion of the particle should be the same in the model as in the prototype and the scales should be chosen accordingly. If this is accepted then Allen's criterion would require $x = y^{3/2}$ or vertical scale equals (horizontal scale)$^{2/3}$. This is considered as an approximate working rule by many Indian River Engineers. The same relationship follows from Lacey's general regime equation,

$$V = 16 \, R^{2/3} \quad S^{1/3}$$

Then
$$V_r = R_r^{2/3} \quad S_r^{1/3} = \frac{R_r^{2/3} \; y_r^{1/3}}{L_r^{1/3}}$$
$$= \sqrt{y_r}$$

Assuming that,

$$R_r \simeq y_r$$

$$\frac{y_r}{L_r^{1/3}} = y_r^{1/2}$$

or
$$y_r = (L_r)^{2/3}$$

6.8. Choice of scales in river models in practice- since the use of light weight material, except for crushed coal, is too expensive to be practical, most of the river models are made of natural materials and do not have true similarity. With proper design and interpretation, they do, however, give valuable information if they have been properly verified. The following discussion will be found useful in deciding scales of natural material models.

(1) Longitudinal Scale : Available discharge and space are often important criteria. Models with a design flow of 0.10 to 0.25 m^3/s are found magnageable in size and yet not too small for useful results.

The width of the model river should not be smaller than about 1 m. This is necessary to study the effects of cut-offs or of river diversions at low stages. If the prototype width of a developing cut-off channel is 15 m., at a scale of say, 1/300, it will be too small in the model.

(2) Depth Scale: For $n_r = 1$, Manning Froude criterion gives

$$y_r = L_r^{3/4}$$

This means use of the same material-when prototype material is sand, this is feasible. For boulder, however, shingle is usually substituted, in which case,

$$y_r = L_r^{3/4} \quad n_r^{3/2}$$

Lacey criterion gives $y_r = L_r^{2/3}$

The above relationships can indicate the range of scale exaggeration.

The scale selected should give (i) a high enough Reynolds no. in the model (ii) a shear Reynolds no. in the required range. In case of sands, the water lepth in the model should be adequate enough to keep the influence of bed ripples at an acceptable level. It may be noted that bed ripples are less prominent in uniform than in graded material. The depth should also not be so high that the bed can not be visually watched.

The scale distortion is related to slope scale and type of material also. General movement should start in the model at a discharge corresponding to that at which it starts in the prototype.

(3) Slope Scale: If no tilt is given $S_r = \frac{y_r}{L_r}$

However, some tilt will often be found desirable in mobile bed channels to increase mobility without increasing distortion.

(4) Discharge scale: Normally, Froude's law for discharge scale is applicable.

$$Q_r = L_r \; y_r^{3/2}$$

Meander length and regime perimeter are proportional to $Q^{1/2}$. According to this $Q_r \propto L_r^2$. If $y_r = L_r^{2/3}$ nearly, this condition will also be satisfied.

Sometimes discharge scale is adjusted by trial and error in the process of verification.

(5) Choice or Material: Bed material should move in the model at the stage corresponding to which active bed movement exists in the prototype. Since large quantities of materials are used in a river model, availability is also a criterion. It is seldom economical to process material for use. For boulder beds, shingle is usually employed. For sandy rivers, sand of the same grade as in the prototype river, or somewhat finer, is usually employed.

(6) Time Scale: For movement of water $t_r = \frac{L_r}{\sqrt{y_r}}$. But for running a hydrograph, generally a time scale has to be evolved by trial. Lower discharges have to be run for relatively longer periods than higher discharges.

(7) Rate of feed of bed material: This is usually adjusted for equilibrium unless effect of aggradation is to be studied.

Table 6.3. gives some of the scales used by I.R.I. Roorkee. The range of vertical exaggeration used at Poona, varies from 3½ to 6 2/3, mostly falling in the range 4 to 6.

Example :- A prototype river has boulder bed and maximum discharge of 10,000 m^3/s. Available space indicates L_r = 1/200. Find other suitable scales.

Using average exaggeration, let y_r be kept 1/50. Then Q_r will be $200 \times 50^{3/2}$ = 1/71,000 and the lab. discharge needed about 0.141 m^3/S. Assuming, this much discharge is available, those scales would be suitable, if minimum width and Reynolds no. are obtained.

Now from equation (12) (for tractive stress ratio)

$$D_r = \frac{y_r^2}{L_r} \quad \text{if unit wt. is the same.}$$

$$= \frac{1}{50 \times 50} \times 200 = \frac{1}{12.5}$$

This is from point of view of mobility.

With this D_r, n_r will be as below :-

$$n_r = (D_r)^{1/6} = \frac{1}{1.525}$$

But to maintain correct flow gauges, required

$$n_r = \frac{y_r^{4/3}}{L_r} = \left(\frac{1}{50}\right)^{4/3} \times 200$$

$$= 1.08$$

This is the contradiction by use of natural material. Flow similarity requires that roughness in the model should be 1.08 times that of the prototype. Mobility requires, it should be $\frac{1}{1.525}$ th. Actually using $D_r = \frac{1}{12.5}$ nearly the roughness can be increased and gauges maintained at correct levels by using wooden pegs or locally placed larger stones.

6.9. Models of Waves:

If the phenomenon to be studied is the form, pattern, and travel of waves (for instance in a study involving break water location for the purpose of reducing wave height) an undistorted model is required. This type of model is designed in strict accordance with the scale relations based upon Froude Law. Ordinarily models with fixed surfaces (usually concrete) are used for this type of study. It may be necessary to adjust the roughness of the model bed to ensure correct velocity of travel of the waves. However it is usually found that the surface roughness has only a small effect, if any, on the wave simulation.

TABLE: 6.3

SOME ACTUAL RIVER MODEL SCALES USED AT I.R.I. ROORKEE

River	Approximate max. discharge m^3/S	d_p mm	d_m mm	L_r	y_r	Vertical exaggeration
Ghagra (Zalimnagar)	28,300	0.27	0.18	250	50	5.0
Sarda (Jaswantnagar)	17,800	0.35	0.18	200	40	5.0
Deoha (Pilibhit)	1,980	0.19	0.18	100	25	4.0
Ganga (Allahabad)	17,000	0.25	0.22	200	32	6.25
Kosi (Kursala)	26,600	0.21	0.23	200	40	5.00
Ganga (Garhmukteswar)	7,350	0.21	0.23	192	36	5.33

The celerity or speed of prepagation of a gravity wave is represented by the expression,

$$C = \sqrt{\frac{g}{2}\,\frac{\lambda}{\pi}\ \tanh\ \frac{2\pi y}{\lambda}}$$

Where C is the celerity, λ the wave length, and y the mean depth.

When $\frac{y}{\lambda}$ is greater than about 0.5, tanh value approaches unity, and $C = \sqrt{g\frac{\lambda}{2\pi}}$. This happens when wave lengths are relatively smaller than the depth. For very long waves in which $\frac{y}{\lambda}$ is 0.1 or less tanh $\frac{2\pi y}{\lambda}$ tends to $\frac{2\pi y}{\lambda}$ and $C \rightarrow \sqrt{gy}$

If scale distortion is used, it should be seen that it does not alter the kind of wave produced.

In harbour models built to investigate wave and surge action, the prototype wave from which protection is desired may range in period from a few seconds to as much as 5 minutes or more. The short period waves (usually from 6 to 18 seconds may range from only a foot or so to more than 20 feet in height. Long waves (usually from about 1 to 5 mts. in period) are generally less than 3 ft. in height. If the problem is primarily one concerning short period waves, an undistorted model should be used. However, as the period of primary waves increases the amount of scale distortion which can be tolerated increases. E.g. a scale distortion of 5 will not result in distorted modes of oscillation in a model when wave periods are about 2 minutes or more.

In models designed to study the stability of rubble mounds or pressures on impervious break waters, an undistorted model should be used and the results interpreted according to Froude's Law.

Where movement of bed material by waves is to be studied, large scale models are to be used. Even so distortion may become necessary. Use of light weight material is often necessary.

6.10. Models of Tidal Areas:

Models for the study of these phenomena are most accurate when constructed to undistorted scales. However if the prototype is wide and shallow or if the study is more concerned with mean velocities rather than with velocity distributions, models with geometric distortion may be used. Scale relations are determined in accordance with principles already described for distorted and undistorted models of open channels.

As large areas are to be reproduced, relatively small scales are common. As much fine sediment is carried, settling velocity criterion is important.

In practice the large discharges involved and small scales used necessitate considerable vertical distortions.

Example:- A model tray 45 m long by 20 m wide is available to make the model of a tidal estuary in which wave formation is not significant. An area 50 km. long and 20 km. wide is to be reproduced in the model which includes 250 sq.km. of sea and creeks. At maximum tidal level, the cross-sectional area of the tidal estuary is 8000 m^2 and the maximum velocity is 1.5 m/S. The tidal range is 4.5 m.

Find the model scales if available discharge is 0.20 m^3/S. Assume that maximum rate of inflow to the sea creeks is 40% more than the average.

To accomodate the model in the tray

$$\text{Length scale} = \frac{45}{50 \times 1000} = \frac{1}{1111}$$

$$\text{or say } \frac{1}{1100}$$

$$\text{Width of tray required} = \frac{20,000}{1100} = 18.2 \text{ m, hence}$$

adequate.

$$\text{Maximum discharge in tidal channel} = 8000 \times 1.5$$
$$= 12,000 \text{ m}^3/\text{S}$$

The tidal cycle is nearly 12.5 hours, with 6.25 hours of tidal rise and 6.25 hours of ebb or fall.

Hence maximum discharge for tidal rise in sea area

$$= \frac{250 \times 1000 \times 1000 \times 4.5}{6.25 \times 3,600} \times 1.40 \quad \text{m}^3/\text{S}$$

$$= 70,000 \text{ m}^3/\text{S}$$

Hence total maximum discharge in prototype is equal to 70,000 + 12,000 = 82,000 m3/S

$$Q_r = \frac{0.20}{82,000} = \frac{1}{4,10,000}$$

$$\text{or } L_r \; y_r^{3/2} = \frac{1}{4,10,000}$$

$$\text{or } y_r^{3/2} = \frac{1}{4,10,000} \times 1100 = \frac{1}{373}$$

$$\text{Hence } y_r = \left(\frac{1}{373}\right)^{2/3} = \frac{1}{52}$$

Tidal rise in model = $\frac{4.5}{52} \times 100$ = 8.65 cms. and hence easily measurable.

$$T_r = \frac{L_r}{\sqrt{y_r}} = \frac{1}{1100}\sqrt{52} = \frac{1}{152}$$

Tidal cycle time in the model $= \frac{12.5 \times 3.600}{152}$

$= 298$ sec.

This is in a suitable working range.

Hence the scales computed above can be adopted.

CHAPTER-VII

MODELS OF SURGE TANK INSTALLATIONS

Let L represent the length of pipe line ab between the reservoir and the surge tank. (Fig.7.1)

- A the sectional area of the pipe line,
- R the ratio of sectional area of the surge tank to that of the pipe line,
- V_1 the velocity in the pipe line before a change of load,
- V_2 the velocity in the pipe line when conditions have settled down after the change of load,
- V the instantaneous velocity in the pipe line during the change
- V' the velocity (simultaneous to V) in the pipe be between the surge tank and the turbines during the change;
- y the height of water surface in the surge tank, above that level which is obtained under steady conditions with initial velocity V_1.

Let us suppose that the loss of head in the main pipe line, of length L, is CV_1^n at velocity V_1 and CV^n at velocity V. This involves a slight approximation.

Now if the suffix m indicates the model and the suffix p indicates the prototype, we must have -

(a) the scale of vertical surges the same as the scale of friction heads, i.e.

$$\frac{y_p}{y_m} = \frac{C_p}{C_m} \frac{V_p^n}{V_m^n} \qquad (1)$$

(b) With initial velocity, V_1, the loss of head in the pipe line ab is CV_1^n and the datum level xx is lower than the reservoir level by this extent. With change of load, the velocity reduces to V and the frictional loss would be CV^n. Under steady conditions the water level would have been lower than the reservoir level by CV^n or higher than the daum xx by $(CV_1^n - CV^n)$. Due to surge the actual level is at a height y above the datum. Hence, head producing retardation at any instant is -

$$y - C(V_1^n - V^n)$$

Force = mass x accn.

$$p_1 \frac{\pi d^2}{4} = -\frac{w}{g} \frac{\pi}{4} d^2 L \frac{dv}{dt}$$

$$p_1/w = h = -L/g \quad dv/dt$$

Consequently $y - C(V_1^n - V^n) = -\frac{L}{g} \frac{dv}{dt} \qquad (2)$

(c) For continuity of flow, the rate of flow into the surge tank, plus that in the pipe between the tank and the turbines must equal that in the main pipe line.

Sectional area of surge tank is AR and sectional area of pipe line is A.

Also let sectional area of pipe line be be R'A.

Then, $(AR)\ \frac{dy}{dt} = AV - AR'V'$

$$\text{or} \quad R\ \frac{dy}{dt} = V - R'V' \qquad (3)$$

If R' is the same in the model and the prototype, the ratio of the right hand side of the equation will be equal to the velocity ratio.

Now for equation (2) to be dimensionally homogeneous, y must have the dimensions of CV^n and of $\frac{LV}{gt}$

$$(1) \qquad \text{Hence} \quad \frac{y_p}{y_m} = \frac{L_p\ V_p}{L_m\ V_m} \cdot \frac{t_m}{t_p} \qquad (4)$$

From equation (1), $\frac{C_p\ V_p^n}{C_m\ V_m^n} = \frac{L_p}{L_m}\ \frac{V_p}{V_m} \cdot \frac{t_m}{t_p}$

$$\text{or} \quad \frac{t_p}{t_m} = \frac{L_p\ C_m\ V_m^{n-1}}{L_m\ C_p\ V_p^{n-1}} \qquad (5)$$

Equation (5) fixes the time scale $\frac{t_p}{t_m}$ for any similar happenings in the model and the prototype.

Again for equation (3) to be homogeneous,

$$\frac{R_p}{R_m} = \frac{V_p\ t_p}{y_p} \cdot \frac{y_m}{V_m\ t_m} \qquad (6)$$

Substituting for $\frac{y_m}{y_p}$ from equation (1) and for $\frac{t_p}{t_m}$ from equation (5) in equation (6)

$$\frac{R_p}{R_m} = \frac{L_p}{C_m} \left(\frac{C_m}{C_p}\right)^2 \left(\frac{V_m}{V_p}\right)^{2n-2} \qquad (7)$$

Equation (7) determines the appropriate size of surge tank for use in Model.

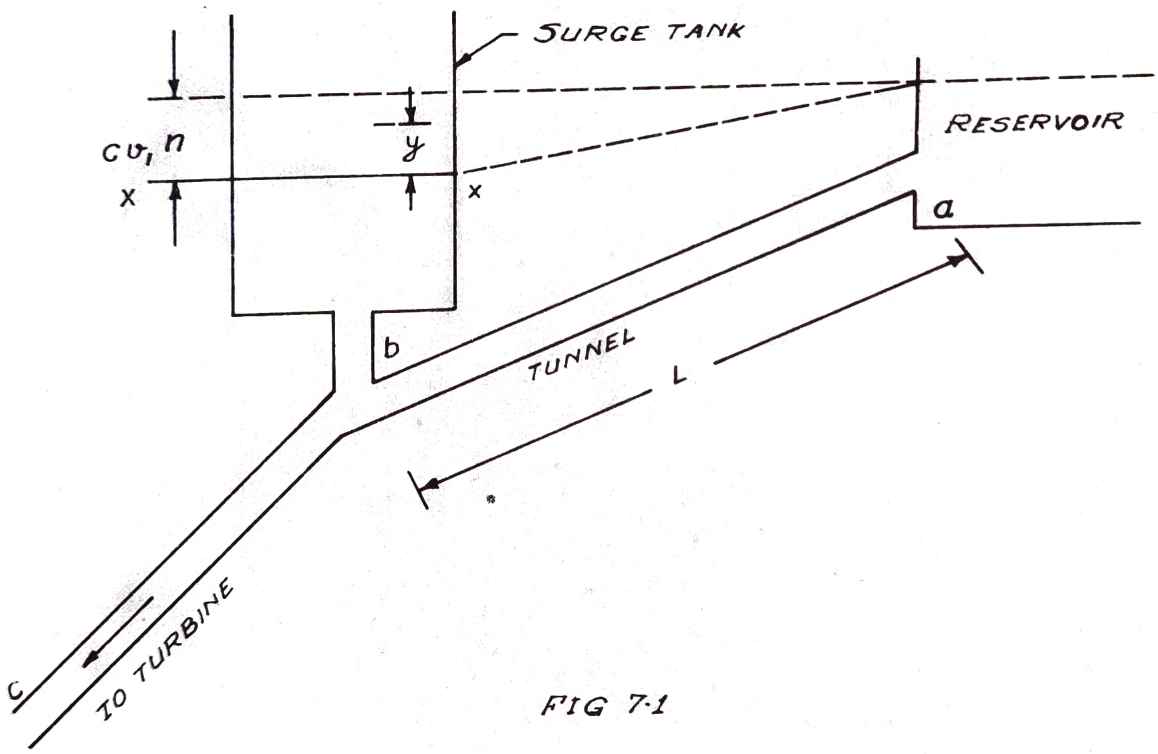

FIG 7·1

Example: A pipe line 150 m long has a diameter of 1.2 m. It is equipped with a surge tank of diameter 3.4 m. The initial flow is at a speed of 1.43 m/s and this is suddenly reduced so as to settle down to 0.58 m/s . The loss of Head in the pipe is 0.18 m at 1.43 m/S.

In constructing a model of this installation, 10 m. long pipe of diameter 7.5. cms is available for which experiment shows a frictional loss of head - $0.75\, V^{1.82}$. Assuming the same index of velocity to apply to the full size pipe, and adopting an initial velocity of 0.3 m/S. for the model determine the final velocity required in the model and the diameter of the model surge tank. What will be the scales of time and of the amplitude of surge ?

$$\text{Velocity ratio} = \frac{0.3}{1.43} = \frac{1}{4.77}$$

Hence final velocity in the model $= \frac{0.58}{4.77} = 0.119$ m/s.

Also $$\frac{y_p}{y_m} = \frac{C_p}{C_m}\frac{V_p^{\,n}}{V_m^{\,n}} = \frac{0.18}{0.75 \times (0.3)^{1.82}} = 7.9$$

But from equation (4)

$$\frac{y_p}{y_m} = \frac{L_p}{L_m}\;\frac{V_p}{V_m}\;\frac{t_m}{t_p}$$

$$\text{or,}\quad 7.9 = \frac{150}{10} \times \frac{1.43}{0.3} \times \frac{t_m}{t_p}$$

$$\frac{t_m}{t_p} = \frac{1}{11}$$

Again from equation (6),

$$\frac{R_p}{R_m} = \frac{V_p\;t_p}{y_p}\;\frac{y_m}{V_m\;t_m}$$

$$= \frac{V_p}{V_m}\;\frac{t_p}{t_m}\;\frac{y_m}{y_p}$$

$$= \frac{1.43}{0.3} \times \frac{1}{7.9}\;\frac{11}{1} = 6.60$$

$$\text{But } R_p = \left(\frac{3.4}{1.2}\right)^2 = 8.05$$

$$\text{Hence } R_m = \frac{8.05}{6.60} = 1.215$$

Dia. of surge tank in model = $\sqrt{1.215}$ x 7.5 = 8.25 cms.

Hence for similarity, if the model were running initially at 0.3 m/sec. it would be necessary to arrange to close it down to a final velocity of 0.119 m/sec. and to provide it with a surge tank 8.25 cms diameter. If then a surge of height x were observed in the model during a time t_m, the anticipated corresponding movement in the prototype would be 7.9 x in a time 11.0 t_m.

CHAPTER-VIII

CONSTRUCTION OF MODELS

8.1. Construction of Models:

Construction of the model is governed by several principles: (i) the model must be an accurate scalar replica of the prototype,

(ii) it should retain its accuracy of construction i.e. it must not deform or settle,

(iii) it should be subject to quick and easy changes in details.

(iv) it must be equipped with appurtenances adequate to ensure control and measurement of flow, observation and measurement, or of photographic recording of pertinent data.

The model must be constructed of materials commonly available, such as wood, concrete, metal, wax, paraffin, plastic, sand and coal. Very frequently, the major part of time spent in a model study is devoted to the construction of the model. Hence available workshop facilities are very important in planning a given study. Proper workshop facilities including wood working, machine and sheet metal shops are a necessary part of any hydraulic model laboratory.

8.2. Horizontal and Vertical Control:

The foundation of accurate model construction lies in the establishment of accurate control nets for both horizontal and vertical dimensions. Details as to horizontal control net generally will depend on the shape and the size of the model. Thus, if the model is long and narrow, the horizontal control may consist of a traverse approximating the centre line of model. If the model is long and wide, the traverse may approximate the model perimeter. If the model covers considerable area, the perimeter traverse may be supplemented by a grid system.

The vertical control system is generally based on a sustantial benchmark located close to the model. If the model covers considerable area supplementary benchmarks may be established. The vertical control net itself is effected by means of an accurate level, great care being taken to attain precision in the levelling operations.

8.3. Elements of the Model:

The essential elements of a model include: (a) facilities for providing the model with a controlled supply of water (b) facilities for insuring natural entrance conditions at the head of the model proper(c) topography, structures to be tested etc. (d) facilities for insuring natural exit conditions at the downstream end of the model and (e) facilities for recording and observing essential data.

Watter Supply:- Water in adequate and controlled quantities must be introduced into the " forebay" of the model. On many hydraulic research stationsin India water is taken from canals and is returned back after reuse. This requires the location of the research station near a canal with one or more falls in the vicinity.

For relatively smaller discharges water can be pumped back after use. In this case sediment trap is necessary to remove the sediment before pumping back the water. For constancy of discharge, normally a constant head tank is needed to supply water to the models.

The size of the supply chamber or forebay depends on the discharge. Baffles and/or screens are provided in the forebay to ensure natural conditions of entry.

8.4. <u>Topography</u>:

Topography in open channel models can be moulded with the help of templets or pegs.

The templets represent prototype sections, reduced to accord with the design scale ratios. The templets are plotted and cut accurately, then located accurately in correct position andat the correct level, and firmly fixed in position. The bed is moulded between successive templets. In this method the templets remain embedded in the model. This would be inconvenient in case of moveable bed models. In that case male templets may be used. The templet support rests on rails set accurately to required grade, and no templets remain embedded (see fig.8.1).

Topography may also be moulded by pegs placed over the model area in such a manner that the tops of the pegs reflect correct topographic elevations. The model surface is then moulded to accord with the tops of pegs.

8.5. <u>Materials for Model Structures</u>:

8.5.1. <u>Wood</u>:- Its dimensions are affected by water content hence it requires water proofing. It is susceptible to warping and is opaque. For water proofing an initial coat of linseed oil, one or more coats of varnish, and a final coat of wax are needed. It is suitable for fabrication of structural supports, and structural elements having plane surfaces or surfaces of straightline generation of short length. Plywood may be bent and warped slightly and thus is suitable for walls that are slightly warped.

8.5.2. <u>Sheet Metal</u>: The material is workable to a limited extent and dimensions are unaffected by water. It tends to buckle under unequal stresses, or in undergoing temperature changes. It is opaque. It may be cut to any shape, but in curves of straight line generation, can not be shaped in warped surfaces, can be fastened metal to wood with counter-sunk screws. Sheet iron must be protected by paint or galvanizing. It is suitable for use as water tight lining of wooden tanks, and for fabrication of structural elements of prismatic shape. It is useful for conduits tubes, gates and walls.

Structural and cast metals are very costly to shape. They are useful for structural supports, model appurtenances etc. A particularly useful material in the laboratory is the slotted angle.

8.5.3. Plastics: A number of plastics are useful in sheet or cast form. Clear transparent sheets of Lucite or Plexiglass may be moulded into intricate shapes in the presence of heat. Joints can be made with suitable solvents like chloroform.

Complicated shapes may be cast using phenol formaldehyde or other resins. The castings can be machined or worked in a manner similar to metals or hard woods. Most of these resins produce opaque castings, but considerable development and some progress is being made in transparent castings.

The strength of these materials varies and they are affected by humidity and temperature and in some cases by sunlight.

Unfortunately the availability of transparent plastic sheets is limited in India these days as they are imported. Transparent pipes and casting resins are not available at all.

Wax: This material is workable, unaffected by water, fragile, susceptible to damage by temperature changes, devoid of structural strength and opaque. It may be poured hot into a mould, cooled and then machined. It is suitable for fabrication of curved model surfaces in which great precision is required.

8.5.4. Masonry, Concrete and Cement Mortar: These materials are very commonly used in India. Where intricate or warped shapes are required masonry or concrete may be used for approximate shaping and final finish can be given in 1:3 cement mortar. The roughness of the surface can be adjusted by varying the size of sand.

These materials are unaffected by water and can be given any desired shape or roughness. However, any alteration of the model requires breaking and rema king of a portion of the model. Pressure points etc. have to be given in advance and holes can not be drilled.

8.6. Appurtenances:

8.6.1. Measurement and Control of Water:

Generally it is necessary to provide the model with a supply of water delivered under a nearly constant head. When the water is drawn from a very large reservoir, the possibility of fluctuations in that supply need not cause concern. If water is pumped, and the voltage of the electricity supply is steady, constant discharge can be assumed;however, voltage flucutations are common in India and directly pumped supply would not be reliably constant. The usual method is to supply the water from a "constant head tank". In this tank the inflow is arranged to be a little more than the outflow and a small flow is always maintained through overflow troughs. A large crest length is provided to the overflow troughs so that a very small head flucutation can take care of inflow or outflow fluctuations.

The measurement of the water supply can be done by a weir on an open flow section, or by a venturimeter, orifice meter or nozzle meter on a pipe line portion. As far as possible all devices should be calibrated after installation.

The advantage of the weir method of measuring discharge is that with one device a large range of flow may be covered. It requires a certain drop of water level for operation. After a change of discharge, sufficient time should be allowed for the gauge to become steady.

The venturimeter and orifice or nozzle meters can be used over limited ranges of flow. They cause very little loss of head.

8.6.2. <u>Measurement of Water Surface</u>:

The water surface level is measured with the help of a pointer gauge or a hook gauge. The pointer gauge can be used directly over the flow or in a gauge well- for a hook gauge a gauge well is usually required. The gaugewell maintains a stillwater surface over which the gauge can be accurately adjusted. Depending on the ratio of the area of the gauge well to that of the connecting openings (with the flow) there will be a time lag between the change of water surface in the flow and the corresponding change in the gauge well. The least count of these gauges is generally, 1/1000 ft. or 0.5 mm.

To obtain a continuous record of fluctuating water levels a " capacitance gauge" is useful. A schematic arrangement for a capacitance gauge is shown in fig.8.2.

The gauge consists of a conductor surrounded by an insulator which is dipped in water. Another conducting probe is also dipped into water. These two are connected to one arm of a wheatstone bridge, the other arms consisting of two known resistances and one known capacitance. The bridge is connected to an A.C. supply. As the water level fluctuates, the capacitance of the gauge changes, and this causes a change of voltage across the terminals AB of the bridge. A continuous record of this variation can be obtained on an oscillograph.

8.6.3. <u>Measurement of Pressure</u>:.

Static pressures are generally measured by piezometers connected to manometer banks. Piezometer tappings should be small, normal to the boundary, and should no project into the flow. The diameter of the opening should be constant for at least three diameters.

For rapidly fluctuating pressures instantaneous values are often required. For these " pressurecells" are used which electrically transmit impulses to a pressure meter where the pressures are recorded on an osillograph film.

8.6.4. <u>Measurement of Current Directions</u>:

(i) Surface currents - the directions of surface currents are obtained by plotting the courses of floats, usually ordinary confetti. By photographing the streak of a float vertically and for a fixed time exposure, a velocity determination is also obtained.

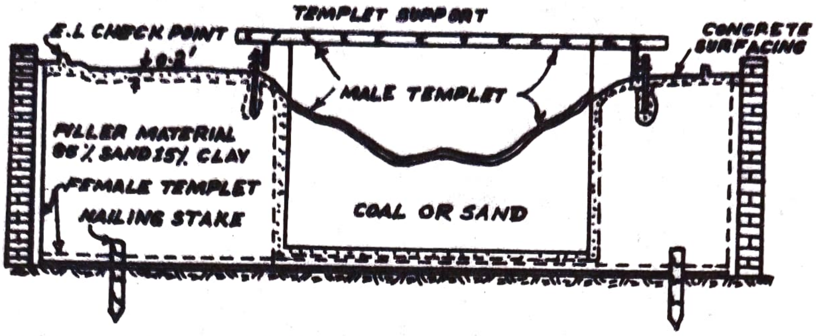

FIG. 8·1 CONSTRUCTION DETAILS OF A MOVABLE BED

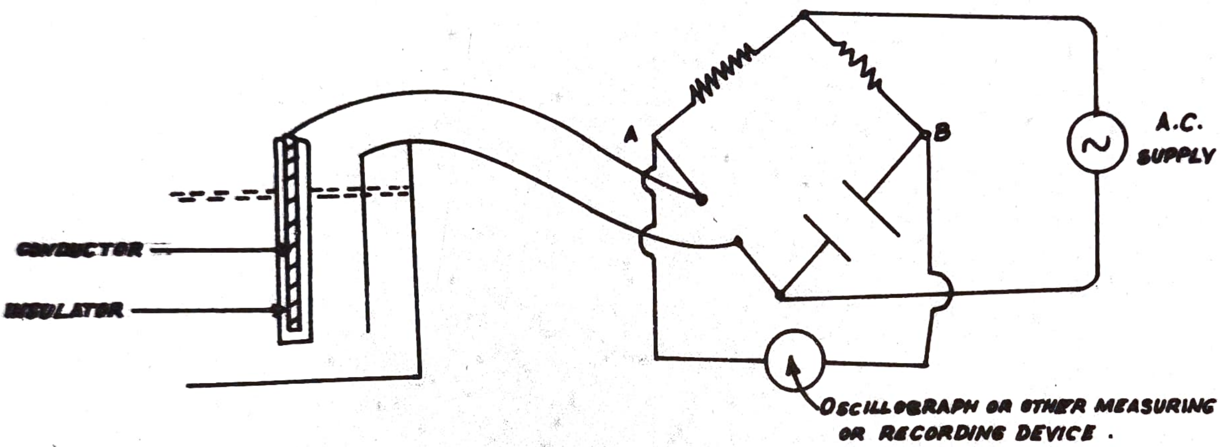

FIG. 8·2 CAPACITANCE GAUGE

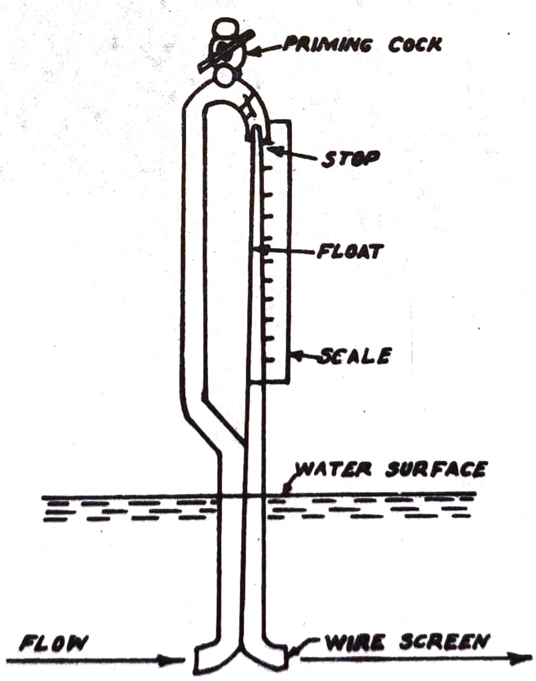

FIG. 8·3 BENTZEL VELOCITY TUBE (MODEL APPURTENANCE FOR MEASURING VELOCITY)

(ii) Bottom Currents - For these bed rollers are used which are just heavier than water- pellets made of creosoted wood, wax, coal etc. serve quite well. " Rattis" where available make excellent bed rollers.

(iii) For intermediate currents a specific gravity of very nearly unity is required. If depth is considerable, hollow balls, partly filled with sand or water, and then sealed can be used.

Immiscible globules having a specific gravity of 1.0 made by mixing benezene with carbon- tetrachloride, or better n. butylthylate and technical xylol, coloured red with an oil dye are very useful.

Short woolen threads attached to wires extending into the flowing water are often used to indicate current directions. Dyes (commonly potassium permaganate or fluorescien) may be used to indicate the general pattern of sub-surface currents by introducing them through small tubes at desired points.

8.6.5. Measurement of Velocity:

Measurement of velocity in models is usually carried out by either of the following three instruments- (i) pitot tube (ii) Midget current meter and (iii) Bentzel tube.

The accuracy of the pitot tube depends on the sensitivity of the manometer used. It is not suited for velocities smaller than about 0.5 ft/sec.

The midget current meter will respond to velocities as small as 0.05 ft/sec.

The Benzel velocity tube (Fig 8.3) will show velocities between 0.15 and 6.0 ft/sec. On account of the pressure difference between the U/S facing and d/s facing tips of the tube, a flow is established through the tube. The equlibrium position occupied by the float depends on the velocity of the main flow. The velocity scale is obtained by actual calibration.

8.6.7. Tail Water Control:

Water surface elevations at lower end of the model must be controlled by an adjustable tail gate weir, or vertical wooden strips, or a valve in the return pipe.

Model Operation:

The operation of the model may be considered in four parts- adjustment verification, tests and preparation of reports.

The process of adjustment consists of testing whether the model is adequate for the purpose for which it was designed. These tests pave the way for the finer adjustments included in the process of model verification.

" verification" process has been discussed earlier in connection with moveable bed models.

8.7. Conduct of Tests:

The procedure involves the model study of a number of alternative designs and the development of the most advantageous design. Great care must be used in conducting and interpreting the tests. While the experimenter must have a thorough knowledge of the fundamentals, great care, ingenuity and patience are also required in correctly interpreting the model results.

The judgement exercised in interpreting model data will often be greatly influenced by the results of the model verification tests. Only if the verification has been clear cut and accurate and if the test occurrences have not differed greatly from the verification occurrences can be experimental data be regarded as quantitatively reliable. A close liasion between the experimenters and the designing engineer is very desirable.

Since most of the expenditure on model study is undertaken in making and verification of the model, there is little economy to be gained in curtailing the programme of tests. During the tests all possibilities, including many not strictly applicable to the problem at hand, may be explored at minor additional expense.

The results of the model investigations should be communicated to the designer continuously in a series of progress reports as action on the works can not often await the preparation of the final comprehensive report.

REFERENCES

(1) 'Hydraulic Models' Manual prepared by A.S.C.E.

(2) 'Fluid Mechanics for Hydraulic Engineers' by Hunter Rouse.

(3) 'Engineering Hydraulics ' Edited by Hunter Rouse.

(4) ' Scale Models in Hydraulic Engineering' by Allen

(5) ' Dimensional Analysis' by Langhar

(6) ' Manual of River Behaviour Control and Training' C.B.I. & P.Publication No.60.

...